Can Ants Design Mechanical Engineering Systems?

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Abstract— The present contribution deals with the optimal tuning of a vibrating blade dynamic vibration absorber by using Ant Colony Optimization (ACO). Dynamic vibration absorbers (DVAs) are systems constituted by mass, spring and damping elements, which are coupled to a mechanical system to provide vibration attenuation. The main idea behind the DVAs is the generation of a force that has the same intensity as the excitation force but in the opposite phase. This phenomenon is known as anti-resonance. The tuning of the DVA is the procedure that sets the anti-resonance frequency to a given value by adjusting the DVA parameters. Based on this theory, the optimization problem is described as the minimization of the objective function that relates the difference between the resonance frequencies of the primary system and those of the DVA. To solve the optimization problem, ACO techniques were used. In the early nineties, when the Ant Colony algorithm was first proposed, it was used as an approach for the solution of combinatorial optimization problems, such as the traveling salesman problem. However, the extension for operating with continuous variables is recent and is still being developed. In this context, this paper presents an engineering application for a continuous domain problem. Numerical results are reported, illustrating the success of using the methodology presented, as applied to mechanical systems.

I. INTRODUCTION

As can be found in [1], in recent years non-gradient based probabilistic search algorithms have attracted much attention from the research community. Even if classical methods (where the search process is guided by information about the gradient) have been widely used due to their computational efficiency, difficulties arise when a local minimum is found. This point is quite often erroneously interpreted as the global optimum. On the other hand, nature-inspired methods are in general zero-order methods, and consequently do not need information about the gradient. They have several advantages, such as easiness to code, robustness to handle different design contexts, efficiency in making use of parallel computing architectures, and a tendency to find the global, or near global, solution. As a consequence of the large number of function evaluations, the disadvantage of facing heavy computational effort can often be observed.

This paper is focused on a recent type of heuristic algorithm, called Ant Colony Optimization (ACO). This method is well established for problems of discrete variables and combinatorial optimization. In the present case, the main contribution is the extension of ACO to optimize continuous design variables as applied to the design of a mechanical system.

ACO, introduced by Marco Dorigo in his doctoral thesis in 1992 [2], is a probabilistic technique for solving computational problems, which can be reduced to finding good paths through graphs. This technique follows some basic concepts, as presented below [3]:

- A search performed by a population of ants, i.e., by simple independent agents.
- Incremental construction of solutions.
- Probabilistic choice of solution components based on stigmergic information of pheromone. A stigmergic process is the process through which the results of a worker insect’s activity act as a stimulus to further activities.
- No direct communication between ants.

II. ANT COLONY FOR CONTINUOUS PROBLEMS

ACO is inspired by the behavior of real ants and their communication scheme by using pheromone trail [4]. A moving ant lays some pheromone on the ground, thus marking the path. The collective behavior that emerges from the participating agents is a form of positive feedback where the greater the number of ants that follow a trail, the more attractive that trail becomes.

A. Algorithm Description

When searching for food, real ants start moving randomly, and upon finding food they return to their colony while laying down pheromone trails [5]. This means that, if other ants find such a path, they return and reinforce it. However, over time the pheromone trail starts to evaporate, thus reducing its attractive strength. When a short and a long path are compared, it is easy to see that a short path gets marched over faster and thus the pheromone density remains high. Thus, if one ant finds a short path (from the optimization point of view, it means a good solution) when
marching from the colony to a food source, other ants are more likely to follow that path, and positive feedback eventually encourages all the ants to follow the same single path.

The idea behind ACO is to mimic this behavior by using artificial ants. The outline of a basic ACO algorithm is as follows:

1. Define the ACO parameters (colony size, initial pheromone trail, dissolving rate)
2. Create an initial colony, randomly distributed throughout the design space (other distributions can be performed)
3. Evaluate the objective function and take it as a path length measure of each ant
4. Perform a complete tour (which mimics the path between the nest and the food source)
5. Update the pheromone trail
6. Repeat steps 2-5 until a stop criterion is met

Fig. 1 ACO basic algorithm.

This way, the first point that has to be taken into account is how to model the pheromone communication scheme. According to [6], for continuous model implementation, this is done by using a normal probability distribution function (PDF) as follows:

\[
\text{pheromone}(x) = e^{-\frac{(x-x_{\text{min}})^2}{2\sigma^2}}
\]  

(1)

where \(x_{\text{min}}\) is the best point found by the optimization task within the design space and \(\sigma\) is an index of the ants aggregation around the current minimum.

“To perform a complete tour,” means a complete update of the values for each design variable for all ants of the colony. Computationally, this can be achieved through a random number generator based on a normal PDF as the variable transition (update) rule to choose the next design variable value of each ant.

Obviously, (1) implies that for each variable update scheme, a different random number generator and its respective PDF are used.

Finally, pheromone distribution over the design space is updated by collecting the information acquired throughout the optimization steps. Since the pheromone is modeled by (1), it is only necessary to update \(x_{\text{min}}\) and \(\sigma\) by:

\[
\sigma = \text{Std}(\text{Colony})
\]  

(2)

where \(\text{Std}(\text{Colony})\) makes use of the colony of ants (candidate solutions) to return a vector containing the standard deviation for each design variable.

Regarding the pheromone scheme, it is possible to see that the accumulation of pheromone increases in the area of the candidate to the optimum. This approach (also called positive update) reinforces the probability of the choices that lead to good solutions. However, to avoid premature convergence, negative update procedures are not discarded, as can be seen in [3].

In this work, a simple method to perform negative update is used, namely by evaporating the pheromone. The idea of this scheme is to spread the amount of pheromone by changing the current standard deviation (for each variable), according to the following equation:

\[
\sigma_{\text{new}} = \gamma \sigma_{\text{old}}
\]  

(3)

where \(\gamma > 1\) is the evaporation rate.

To initialize the algorithm:
1. \(x_{\text{min}}\) is randomly chosen within the design space using a uniform PDF;
2. \(\sigma\) is taken as three times greater than the length of the search interval.

Differently from both the Genetic Algorithms (GA) and the Particle Swarm Optimization (PSO) methods, which have a set of parameters to be defined by the user, ACO has only a single special parameter to be chosen, namely the evaporation rate \(\gamma\).

A comparison between nature and ACO algorithm can be seen in Table 1:

<table>
<thead>
<tr>
<th>Nature</th>
<th>ACO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible paths between the nest and food</td>
<td>Set of possible solution (vector of design variables)</td>
</tr>
<tr>
<td>Shortest path</td>
<td>Optimal solution</td>
</tr>
<tr>
<td>Pheromone communication in action</td>
<td>Optimization procedure</td>
</tr>
</tbody>
</table>

B. Stop Criteria

All tests in this work repeat an iterative optimization task until at least one of the stop criteria is achieved. The stop criteria implemented in the algorithm are shown in Table II.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterations</td>
<td>Fixed number of generations reached</td>
</tr>
<tr>
<td>TimeLimit</td>
<td>The total time (in seconds) allowed for the optimization task.</td>
</tr>
<tr>
<td>StallIterLimit</td>
<td>If after this number of iterations there is no improvement, the simulation will abort.</td>
</tr>
<tr>
<td>StallTimeLimit</td>
<td>If after this period of time (in seconds) there is no improvement, the simulation will abort.</td>
</tr>
</tbody>
</table>
III. APPLICATION TO THE DESIGN OF MECHANICAL SYSTEMS

Dynamic vibration absorbers (DVAs) are systems constituted by mass, spring and damping elements (secondary structure), which are coupled with a mechanical system (primary structure) in order to attenuate the vibrations of the primary structure in a given frequency range, [7] and [8]. The main idea behind the DVA is the generation of a force that has the same intensity as the excitation force but in the opposite phase. This phenomenon is known as anti-resonance. The tuning of the DVA is the procedure that sets the anti-resonance frequency to a given value by changing the DVA parameters (mass, spring and damping values). Now, let’s consider the vibrating system characterized by a two degree of freedom (D.O.F) structure, composed by an undamped primary structure \((m_1, k_1)\) and a damped secondary structure (the DVA), \((m_2, k_2\) and \(c_2\)), as illustrated in Fig. 2.

The equations of motion for the system above are given by:

\[
\begin{align*}
m_1 \cdot \ddot{x}_1(t) + k_1 \cdot \ddot{x}_1(t) + k_2 \cdot (x_1(t) - x_2(t)) + &\quad c_2 \cdot (\dot{x}_1(t) - \dot{x}_2(t)) = P_1 \cdot \sin(\omega \cdot t) \\
m_2 \cdot \ddot{x}_2(t) + k_2 \cdot (x_2(t) - x_1(t)) + &\quad c_2 \cdot (\ddot{x}_2(t) - \ddot{x}_1(t)) = 0
\end{align*}
\]

(4)

From (4) it is possible to obtain the non-dimensional equation for the vibration amplitude of mass \(m_1\):

\[
H(g) = \frac{\left(\delta^2 - g^2\right) + j\delta^2 \xi g}{\left(1 - g^2\right)\left(\delta^2 - g^2\right) + j\delta^2 \xi g + \mu \left(j\delta^2 \xi g^3 - \delta^2 g^3\right)}
\]

(5)

where \(\mu = \frac{m_2}{m_1}\), \(\omega_n^2 = \frac{k_1}{m_1}\), \(\omega_2^2 = \frac{k_2}{m_2}\), \(\delta = \frac{\omega_2}{\omega_n}\), \(g = \frac{\omega_2}{\omega_n}\), and \(\xi = \frac{c_2}{2m_2\omega_n}\).

Reference [8] defines the optimal DVA as the one that causes the frequency response function (FRF) to have the same response for both the points P and Q (called invariant points). This can be achieved by handling the relation between the natural frequencies, \(\delta\), and the damping factor \(\xi\). Fig. 3 shows the graphics of the vibration amplitudes, adopting \(\mu = 0.05\) and \(\delta = \delta_{\text{OPTM}} = \frac{1}{1 + \mu}\), for different values of \(\xi\).

To close the review about DVA, for primary structures in which \(N\) modes must be attenuated, \(N\) DVAs, each one tuned to a specific mode, have to be attached to the primary structure. Attenuation is obtained at the attachment points. This configuration is called the Multi-Mode Dynamic Vibration Absorber (MMDVA). In this case the various DVAs cannot be designed separately, using closed solutions, as in the single mode case [9]. Thus, the most effective strategy should include both the influence of each mode and the frequency range of interest. In this work, the design of the MMDVA is treated as an optimization problem, as presented in [10] and [11].

![Fig. 2. Undamped primary structure and damped DVA.](image)

![Fig. 3. Vibration amplitude of the primary mass with a damped DVA.](image)

IV. FINITE ELEMENT ANALYSIS OF THE SYSTEM

Fig. 4 shows the mechanical system studied in this work. Fig. 4-(a) illustrates the primary structure (a clamped-clamped beam) and the MMDVA. Figures 4-(b) and (c) give details about the primary structure and the shape of the MMDVA.
Some remarks about the MMDVA must be taken into account. Each double blade (five in this case) behaves like a single DVA. These DVAs are not independent, since the addition of an extra DVA can affect the behavior of the structure as a whole. This way, each DVA can be tuned by the correct choice of its geometry, according to a proper optimization process. In addition, in order to increase the amount of damping of the MMDVA, the whole structure should be immerged in a fluid or, alternatively, some viscoelastic material should be bonded to the surface of each blade [12].

A. Finite Element Model of the Primary Structure

Fig. 5 shows the primary structure: Fig. 5-(a) shows the mechanical arrangement; Fig. 5-(b) and (c) give details of the finite element model.

Table III gives the material properties of the finite element model of the primary structure.

By performing the modal and harmonic analysis of the primary structure, it is possible to obtain the natural frequencies and the frequency response function (FRF) of the system. Fig. 6 shows the FRF of the primary structure. Table IV gives the values of the natural frequencies of the primary structure. All analyses were performed by using the commercial software ANSYS®.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Natural frequencies [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2095.5</td>
</tr>
<tr>
<td>2nd</td>
<td>2684.1</td>
</tr>
<tr>
<td>3rd</td>
<td>3352.4</td>
</tr>
<tr>
<td>4th</td>
<td>4094.8</td>
</tr>
<tr>
<td>5th</td>
<td>4911.2</td>
</tr>
</tbody>
</table>

B. Finite Element Model of the MMDVA

Fig. 7 illustrates the finite element model and Table V gives the material properties of the MMDVA.

Table V

<table>
<thead>
<tr>
<th>Young’s modulus</th>
<th>Poisson ratio</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.06 \times 10^3$ GPa</td>
<td>0.3</td>
<td>7850 kg/m$^3$</td>
</tr>
</tbody>
</table>
V. DESIGN STRATEGY

As previously discussed, for design purposes the most effective strategy should consider the influence of each mode and the frequency range of interest. The FRF is to be minimized over predefined frequency bands as chosen by the user. For example, in the case illustrated by Fig. 6, five modes were taken as target modes for vibration attenuation. Consequently, in the most general case, the system may have a set of DVA’s, each one tuned up for a particular mode. Under these circumstances, the optimal design of MMDVA consists in determining the value of each one of the blade lengths and their corresponding damping factors.

Then, two formulations for the optimization problems were tested. The first one, which considers the neighborhood of each target mode, is defined as the minimization of the objective function given by:

\[
J \{ X \} = \sum_{i=1}^{5} \max_{f \in \{ \text{modes} \}} \{|H(f)|\}
\]  

Subject to:

\[
X_1 > X_2 > X_3 > X_4 > X_5
\]
\[
5 \times 10^{-3} \leq X_i \leq 15 \times 10^{-3}
\]
\[
i = 1,2,3,4,5
\]
\[
0.005 \leq X_6 \leq 0.025
\]  

where:

• \(X_1\) to \(X_5\) are the lengths of the blades of the MMDVA (given in meters).

• \(X_6\) is the non-dimensional damping factor of each blade of the MMDVA (damping is considered the same for all the blades).

As the design requires \(X_1 > X_2 > X_3 > X_4 > X_5\), gives rise to a pine-shaped MMDVA. This procedure also reduces the redundancies of the optimal design.

Table VI gives the values of the bounds used in the optimal tuning of the MMDVA for the present formulation.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Lower bound [Hz]</th>
<th>Upper bound [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2000</td>
<td>2198</td>
</tr>
<tr>
<td>2nd</td>
<td>2582</td>
<td>2786</td>
</tr>
<tr>
<td>3rd</td>
<td>3248</td>
<td>3458</td>
</tr>
<tr>
<td>4th</td>
<td>3992</td>
<td>4196</td>
</tr>
<tr>
<td>5th</td>
<td>4808</td>
<td>5000</td>
</tr>
</tbody>
</table>

The second formulation, which considers the whole frequency band, is defined as the minimization of the objective function given by:

\[
J \{ X \} = \max_{2000 \leq f \leq 5000} \{|H(f)|\}
\]  

Subject to:

\[
X_1 > X_2 > X_3 > X_4 > X_5
\]
\[
5 \times 10^{-3} \leq X_i \leq 15 \times 10^{-3}
\]
\[
i = 1,2,3,4,5
\]
\[
0.005 \leq X_6 \leq 0.025
\]  

VI. RESULTS

As presented above, the first optimization problem formulation considers the neighborhood of each mode to be controlled and the second one is devoted to the whole frequency band studied.

Table VII shows the setup for the ACO used for both design approaches.

<table>
<thead>
<tr>
<th>Number of ants</th>
<th>Maximum number of iterations</th>
<th>Evaporation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Fig. 8 shows the FRFs obtained for the uncontrolled and controlled cases, according to the first formulation. Fig. 8-(a) presents the regions of the frequency spectrum for which vibration amplitudes are to be minimized. Figures 8-(b) to (e) illustrate the evolution of ACO during the optimization process. Fig. 8-(b) shows how the best ant, the mean and the standard deviation of the colony evolve during the optimization procedure. Fig. 8-(c) gives the histogram of the objective function values for the colony at the end of the optimization run. Fig. 8-(d) illustrates how the colony is dispersed along the iterations. Finally, Fig. 8-(e) indicates the criterion achieved to stop the optimization process.

Fig. 9 shows the FRFs obtained for the uncontrolled and controlled cases, according to the second formulation proposed. Fig. 9-(a) presents the regions of the frequency spectrum for which vibration amplitudes are to be minimized. Figures 9-(b) to (e) show the evolution of ACO during the optimization process.
VII. CONCLUSIONS

The present contribution was dedicated to the study of the continuous Ant Colony Optimization (ACO) techniques applied to the optimal design of a multimode dynamic vibration absorber (MMDVA). As ACO for continuous problems is still under development, this paper presents a contribution in the sense of exemplifying an application of continuous ACO in the field of Mechanical Engineering. The methodology was able to handle six continuous design variables in the optimization of a MMDVA. As no closed
solution was available, it was shown that for the multi-mode damper the optimal MMDVA parameters had to be obtained by using optimization techniques. It is worth mentioning that the multi-mode technique is necessary in a great number of real situations, in which the natural frequencies are close together in the frequency spectrum. Finally, the authors are encouraged to pursue their research effort in the sense of: 1- comparing the performance of ACO with other heuristic optimization algorithms (such as Genetic Algorithms and Particle Swarm Optimization), and 2- trying to solve complex engineering problems by using nature-inspired optimization approaches.

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