Implementing Multi-Modal Shunted Piezoelectrics

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ABSTRACT

It is known that shunted piezoelectrics technology can be used to dissipate part of the vibration energy. In this sense, a multimodal shunted piezoelectric is a passive damping device that allow to add damping over a frequency band of the structure. The device is compound by piezoelectric elements and proper RL networks (since large inductances are required, synthetic inductors are commonly used). From the electrical viewpoint, the piezoelectric elements behave like a capacitor in series with a controlled voltage source and the corresponding shunt circuits can be tuned to dissipate electrical energy over a given frequency band. This work is devoted to the study of passive damping systems for multiple modes, based on piezoelectric patches and resonant shunt circuits. The present contribution discusses the modeling of piezoelectric patches coupled to shunt circuits, where the basics of resonant shunt circuits are presented. Following, the devices used in passive control are analyzed from the electrical and experimental viewpoints. It is defined a design methodology for multi-modal systems by using optimization techniques. Finally, experimental results are reported, illustrating the success of using the methodology presented in passive damping applications applied to mechanical and mechatronic systems.

Keywords: multi-modal damping, shunted piezoelectric, resonant shunt circuits.

Nomenclature

PZT is the piezoelectric patch, 
\( i \) and \( j \) give the coordinate axes that indicate the direction for electrical and mechanical vectors, 
\( T_j \) is the stress \([\text{N/m}^2]\), 
\( A_i \) and \( A_j \) are the transversal section areas according to the vectors \( i \) and \( j \) \([\text{m}^2]\), 
\( B_i \) and \( B_j \) are the initial patch lengths according to vectors \( i \) and \( j \) \([\text{m}]\), 
\( S_j \) is the strain (non-dimensional), \( Z_i^{\text{SHUNT}}(s) \) is the electrical impedance of the shunt circuit \([\Omega]\), 
\( s \) is the Laplace variable, 
\( D \) is the vector of electrical displacement \([\text{C/m}^2]\), 
\( E \) is vector of electric field \([\text{V/m}]\), 
\( S \) is the vector of material engineering strains (non-dimensional), 
\( T \) is vector of material stresses \([\text{N/m}^2]\), 
\( \varepsilon \) is the matrix of dielectric constants for the material \([\text{C}^2/(\text{N.m}^2)]\), 
\( s \) is the matrix of compliance for the material \([\text{m}^2/\text{N}]\), 
\( d \) is the piezoelectric material constant relating strain to voltage \([\text{m/V}]\). 
\( k_r \) and \( m_r \) are commonly known as the modal stiffness and modal mass, respectively (for mode \( r \)), 
\( Z_{jj}^{\text{MEC}}(s) \) is the mechanical impedance associated with the resonant shunted piezoelectric,
Modeling of Piezoelectric Patches coupled to Shunt Circuits

Figure 1 illustrates the case of a piezoelectric device (in the patch form) coupled to a shunt circuit and bonded to a flexible beam-like structure. Figure 1-(a) and (b) show the complete system and the simplified model for the passive damping system, respectively.

As in [1], the general expression that describes the behavior of piezoelectric materials, such as the one shown in Figure 1, is written as:

\[
\begin{bmatrix}
D \\
S
\end{bmatrix} =
\begin{bmatrix}
\epsilon^T & d \\
d^T & s
\end{bmatrix}
\begin{bmatrix}
E \\
T
\end{bmatrix}
\]  

(1)

The modeling for multi-modal shunted piezoelectrics starts from the one for single mode. This way, according to [1], [3] and [7], the frequency response function (FRF) that relates the acceleration and the external force of a resonant shunted piezoelectric in the Laplace domain is:

\[
H(s) = \frac{s^2}{k_r + sZ_{MEC}^{ij}(s) + s^2 m_r}
\]

(2)

Figure 2 shows the circuit used in this work, i.e., the so-called resonant shunt circuit in the parallel topology.

Previous contributions such as [1], [2] [3] and [4] show how to obtain the expression of the mechanical impedance of the shunted piezoelectric. Thus:

\[
Z_{MEC}^{ij}(s) = Z_{ij}^D(s) \frac{1 - k_{ij}^2}{1 - k_{ij}^2 \epsilon_i^{ELECT}(s)}
\]

(3)
where:

- \( Z_{ij}^D(s) = \frac{K_{ij}^E}{s(1-k_{ij}^E)} \) is the mechanical impedance of the open circuit piezoelectric,
- \( K_{ij}^E \) is the mechanical stiffness of the shunted piezoelectric,
- \( k_{ij} = \frac{d_{ij}}{\sqrt{E_{ij}^T E_{ij}}} \) is the electromechanical coupling coefficient (physically, its square value represents the percentage of mechanical strain energy that is converted into electrical energy and vice-versa),
- \( Z_{ij}^{ELECT}(s) = \frac{sC_{PZT}^T}{sC_{PZT}^T + Y_{SHUNT}} \) is the non-dimensional electrical impedance,
- \( C_{PZT}^T \) is the capacitance of the piezoelectric patch at constant stress (free), and
- \( Y_{SHUNT}(s) = \frac{1}{Ls} + \frac{1}{R} \) is the electrical admittance of the shunt circuit.

Other shunt circuit configurations have been investigated, such as the resistive (which behaves similar to viscoelastically-damped systems), switched (which adjusts the behavior of the circuit in response to any change in the system.), and capacitive (which changes the stiffness of the piezoelectric element), as discussed in [5]. More recently, an arrangement with a negative capacitance and a common resistor has been proposed in [6]. This scheme works in the sense of canceling the capacitance of the piezoelectric element in such a way that all the electrical energy is dissipated through the Joule effect.

For the MDOF system, the FRF that relates the acceleration obtained at the position \( i \) when an external force is applied at the position \( k \) is given by the following equation:

\[
H_{ik}(s) = \sum_{r=1}^{N} s^2 \frac{\psi_{ir} \psi_{kr}}{k_r + s^2 m_r}
\]

where each pair \( \{\psi_r\} \) and \( \omega_r = \sqrt{\frac{k_r}{m_r}} \) characterizes a mode of vibration of the system. In order to obtain more details about MDOF systems refer to [7].

It is possible to note the similarity between Eq. (2), which describes the system of 1 DOF containing the passive damping device and Eq. (4), which describes a MDOF system. It can be observed that Eq. (4) takes into account the influence of the N modes on the system response.

This way, by considering the analyses above, the effect of the piezoelectric patches and their shunt circuits can be introduced easily in the MDOF system. This can be achieved by adding the term \( Z_{ij}^{MEC}(s) \) in the denominator of Eq. (4). Mathematically:

\[
H_{ik}(s) = \sum_{r=1}^{N} s^2 \frac{\psi_{ir} \psi_{kr}}{k_r + sZ_{ij}^{MEC}(\omega) + s^2 m_r}
\]

**Electric Analysis of the Shunted Piezoelectric Components**

As can be viewed in the experimental section, in the real cases, the associated frequencies are relatively low. As a consequence, the resonant shunt circuits require large values for the inductance. These values would typically attain hundreds of henries. The weight and the volume of such inductors would make unfeasible the use of this technique. To overcome this limitation, synthetic inductors are obtained by using operational amplifiers, as in previous works ([10], [9] e [11]). Even assuming different configurations, these circuits are known as synthetic
inductors or gyrators. In the present contribution, two types of synthetic inductors were explored, namely the one proposed by [10] and the other one based on [9]. For the sake of simplicity, they are called Antoniou synthetic inductor and Riordan synthetic inductor, respectively.

**Antoniou and Riordan Synthetic Inductors**

Figure 3 shows the synthetic inductor circuits.

![Figure 3 – Circuit: (a) Antoniou synthetic inductor. (b) Riordan synthetic inductor.](image)

In despite of presenting a different topology, the equations for the input impedance are the same for both synthetic inductors. The input impedance is given by equation (6):

\[
Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}
\]

(6)

From the previous equation, a synthetic inductor is obtained by using the following relations: \( Z_4 = \frac{-j}{\omega C_4} \), \( Z_1 = R_1 \), \( Z_2 = R_2 \), \( Z_3 = R_3 \) and \( Z_5 = R_5 \). The equivalent circuit impedance \( Z_{IN} \) is the same as an inductor \( L_{eq} \), as shown below:

\[
Z_{IN} = j \omega L_{eq};
\]

\[
L_{eq} = \frac{R_1 R_3 C_4 R_5}{R_2}
\]

**Practical Aspects about the Synthetic Inductors**

In order to test the performance of synthetic inductors at low electrical frequencies (as typically found in most mechanical systems), experiments simulating the resonant shunt circuit were performed. An RLC series filter was configured in this experiment by using the synthetic inductor together with a resistor \( R_0 \), a capacitor \( C_{PST} \) and a signal generator (Brüel & Kjaer Sine/Noise Generator Type 1049). A signal analyzer (Spectral Dynamics SD380) was used to perform the frequency analysis of the circuit. The setup of the experiment is shown in Figure 4.
The RLC filter is described by Eq. (8), which relates the voltage in \( R_0 \) and the voltage in the generator.

\[
H_{LOAD}(\omega) = \frac{1}{1 + j \frac{1}{R_0} \left( \frac{\omega L}{\omega C_{PZT}} - \frac{1}{\omega C_{PZT}} \right)}
\]  

(8)

The signal generator introduces in the circuit a white noise of bandwidth 2Hz to 2kHz, with 1.25V of RMS value. The signals were acquired simultaneously in a sample of \( T = 1.6s \), with intervals of \( dt = 0.78125\text{ms} \). Then the maximum frequency analyzed is \( f_{\text{max}} = 500\text{Hz} \) and the frequency resolution is \( df = 0.6249\text{Hz} \). The transfer function was estimated by using 50 samples.

The capacitors \( C_4 \) and \( C_{PZT} \) and the resistor \( R_0 \) were fixed to 108.5 nF, 110.1 nF and 9.98 k\( \Omega \), respectively. Table 1 shows the values used for the remaining components along the experiments. In this case, the inductance is calculated according to the equation:

\[
L_{eq} = \frac{1}{(2\pi f_{\text{det}})^2 C_{PZT}}
\]  

(9)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( R_1 ) [k( \Omega )]</th>
<th>( R_2 ) [k( \Omega )]</th>
<th>( R_5 ) [k( \Omega )]</th>
<th>( R_3 ) [k( \Omega )]</th>
<th>( L ) [H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2.18</td>
<td>2.17</td>
<td>97.7</td>
<td>2.16</td>
<td>23.00</td>
</tr>
<tr>
<td>#2</td>
<td>5.51</td>
<td>5.52</td>
<td>38.6</td>
<td>5.51</td>
<td>23.03</td>
</tr>
<tr>
<td>#3</td>
<td>14.78</td>
<td>14.68</td>
<td>14.35</td>
<td>14.67</td>
<td>22.0</td>
</tr>
<tr>
<td>#4</td>
<td>46.2</td>
<td>46.2</td>
<td>4.61</td>
<td>46.0</td>
<td>23.01</td>
</tr>
<tr>
<td>#5</td>
<td>99.9</td>
<td>99.3</td>
<td>2.13</td>
<td>99.1</td>
<td>23.04</td>
</tr>
<tr>
<td>#6</td>
<td>219</td>
<td>220</td>
<td>0.964</td>
<td>221</td>
<td>23.01</td>
</tr>
<tr>
<td>#7</td>
<td>330</td>
<td>329</td>
<td>0.643</td>
<td>329</td>
<td>23.02</td>
</tr>
</tbody>
</table>

Figure 5 presents the graphics of the amplitude, phase and coherence of the transfer function for the two types of synthetic inductors. A comparison between the two types of synthetic inductors does not allow any straightforward conclusion about the superiority of one with respect to the other. However, by taking into account the graphics of the module and phase, it is possible to notice good repeatability for the tests, since the resonance frequency remains approximately the same for all the experiments. The graphics of the coherence indicate that along the experiments the values of the coherence become worse, probably due to the values of the resistors. Finally, it is also possible to notice that the electric network available contaminates the experiments in 60Hz (used in Brazil) and in some other harmonics in all the experiments.
Figure 5 also shows a discrepancy between the experimental results and the analytical model adopted for the RLC circuit, as described by Eq. (8). For example, in all cases the gains are not unitary in the resonant frequency. It can be concluded that the non-idealities of the synthetic inductor generate one or more parasite resistances associated with the equivalent inductor. A previous work, [12], discussed about an experiment that evaluates the impedance of the synthetic inductor. By using an impedance analyzer, it was shown the existence of a frequency dependent inherent resistance in the synthetic inductor.

![Figure 5 - Amplitude, phase and coherence of the transfer function for Antoniou and Riordan synthetic inductors.](image1)

In this work, it was obtained an equivalent model for the synthetic inductor, in which it is characterized as a resistor $R_{\text{PARASITE}}$ in series with an equivalent inductor $L_e$, as illustrated by Figure 6.

![Figure 6 - Synthetic inductor equivalent circuit: a resistor $R_s$ in series with an inductor $L_{eq}$.](image2)

Once defined the model for the synthetic inductor, a second set of experiments verifies the behavior of the inherent resistance as a function of the value of the equivalent inductor, both for the Antoniou and Riordan synthetic inductors. In the experiments, the values of $R_0$ and $C_0$ were fixed to 9.86 kΩ and 111.5 nF, respectively, and a number of different values for the resonance frequency of the circuit, $f_0$, was predefined. Consequently,
the value of the inductor varies, leading to the variation of $R_{\text{PARASITE}}$. For both considered inductors the values of $R_0$, $R_1$, $R_2$, $R_5$, $C_0$, and $C_4$ were fixed to 9.86k$\Omega$, 46.2k$\Omega$, 46.2k$\Omega$, 46.0k$\Omega$, 111.5nF, and 112.6nF, respectively. The results can be seen in Figure 7.

![Figure 7](image)

**Figure 7 - $R_{\text{PARASITE}}$ versus $L$ for the synthetic inductors.**

Both for the Antoniou and for the Riordan synthetic inductors, it is easy to notice that the value of the parasite resistance increases when the inductance values also increase. This behavior is quite linear along the analyzed range. This means that, by taking into account a pre-defined value for the capacitance, the RLC filters exhibit better performance when designed to operate at larger electrical resonance frequencies. This fact has a direct consequence in shunted piezoelectric applications, since the frequencies of interest in these cases are rather low from the electrical point of view.

**Design Methodology for Multi-Modal Systems**

For illustration, consider the situation shown in Figure 1-(a) and described by Eq. (5). The response, $H_{ik}(s)$, must be minimized over predefined frequency bands as chosen by the user. In this example, the second and third modes were taken as target modes for attenuation purposes. Consequently, in the most general case, the system may have a group of $N = 2$ piezoelectric patches and their shunt circuits, each one tuned up for a different mode. Under these circumstances, the optimal design of shunt circuits consists of determining the value of each one of the $N = 2$ inductors and $N = 2$ resistors to be used in the shunt circuits.

Then, the optimization problem is defined as the minimization of the objective function given by:

$$ J(\{L\}, \{R\}) = \sum_{m=1}^{p} |H_{ik}(s_m)| $$

subject to:

$$ L_i^{\text{lower}} < L_i < L_i^{\text{upper}}, \quad R_i^{\text{lower}} < R_i < R_i^{\text{upper}}, \quad i = 1, 2, ..., N $$

where:

- $p$ : is the number of values of $H_{ik}(s)$; these values are computed by using the modeling of multi-modal piezoelectric shunt systems as shown in the previous section,
- $H_{ik}(s)$ : is the frequency response function,
- $L_i$ and $R_i$ : are the inductors and resistors to be used in the damping of the $ith$ mode, respectively, and
- $N$ : is the number of shunt circuits to be used.
The optimization task that defines the design of the shunt circuits is an example of direct problem. In spite of this, the use of classical optimization methods fails in many cases, due to local minima found in the design space. For this reason, in the present work, a natural optimization method was used, namely the LifeCycle Model, to be briefly reviewed in the following sub-section. In order to obtain details about LifeCycle Model consult [3] and [8].

Figure 8 shows the FRF obtained numerically. The circuits were designed to reduce the vibration amplitudes of the second and third modes, simultaneously.

![Figure 8 - FRF for systems with and without passive damping.](image)

**Experimental Results**

Using a mechanical system composed by a flexible beam, two piezoelectric patches and their respective shunt circuits, it was possible to perform the experimental verification of the design methodology presented above. Figure 9 shows details of the experimental setup.

![Figure 9 – Experimental apparatus.](image)

As described previously, the design of shunt circuit parameters for the multimode case is treated as an optimization problem. The attenuation of the vibration in the neighborhood of the second and third modes is addressed in the present work.
Table 2 gives the shunt circuit parameters, as obtained by following the optimization approach as described above. Figure 10 gives the characteristics of the optimal design. Figure 10-(a) shows the frequency bands of the spectrum chosen to have their vibration amplitudes minimized. Figure 10-(b) and (c) give the evolution of the LifeCycle during the optimization process.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$R_{OPTM}$ [KΩ]</th>
<th>$L_{OPTM}$ [H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd</td>
<td>814.8445</td>
<td>17.1468</td>
</tr>
<tr>
<td>3rd</td>
<td>53.7718</td>
<td>2.5338</td>
</tr>
</tbody>
</table>

Figure 10 - Design for second and third modes.

Figure 11 shows the experimental results obtained for the multimode case.

Figure 11 - Experimental results for the first and second modes.

Conclusions

The present contribution was dedicated to the study of passive vibration damping by using piezoelectric patches and resonant shunt circuits. The analytical model of the system shows that the general behavior of shunted piezoelectric systems is similar to the classical dynamic vibration absorber. It is known that closed form solutions for the shunt parameters in the case of single mode attenuation are available. However, in the case of the multi-mode damper, the optimal shunt parameters have to be obtained through optimization techniques. An important aspect is that the multi-mode technique is necessary in real life applications, particularly when the modes are quite close in the frequency spectrum. In such cases the influence of each mode in the response cannot be discarded. It was necessary to design synthetic inductors based on operational amplifiers, to avoid cumbersome
traditional inductors. Both numerical and experimental results are very encouraging in the sense that electric shunt circuits can be successfully used in order to attenuate vibrations of flexible structures.

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References