ABSTRACT
To achieve high efficiency, modern gas turbines operate at temperatures that exceed melting points of metal alloys used in turbine hot gas path parts. Parts exposed to hot gas are actively cooled with a portion of the compressor discharge air (e.g., through film cooling) to keep the metal temperature at levels needed to meet durability requirements. However, to preserve efficiency, it is important to optimize the cooling system to use the least amount of cooling flow. In this study, film cooling optimization is achieved by varying cooling hole diameters, hole to hole spacing, and film row placements so that the specified targets for maximum metal temperature are met while preserving (or saving) cooling flow. The computational cost of the high-fidelity physics models, the large number of design variables, the large number and nonlinearity of responses impose severe challenges to numerical optimization. Design of experiments and cheap-to-evaluate approximations (radial basis functions) are used to alleviate the computational burden. Then, the goal attainment method is used for optimizing of film cooling configuration. The results for a turbine blade design show significant improvements in temperature distribution while maintaining/reducing the amount of used cooling flow.

1 INTRODUCTION
Thermodynamic efficiency of industrial gas turbines increases with turbine inlet temperature. In fact, in most modern gas turbines, the temperature of the combustion gas exceeds the melting point of the alloys used. Parts exposed to the hot gas are actively cooled with a portion of compressor discharge air (keeping metal temperatures at levels needed to meet durability targets).

One example of such cooling system is a technology called film cooling [1]-[3]. Film cooling involves forming a blanket of air at the surface of the part; reducing the temperature the part is exposed to. Unfortunately, the use of compressor discharge air reduces efficiency of the gas turbine (because air used for cooling is not performing work). Film cooling design involves generation of a practical layout of film cooling holes [4] that makes the part able to maintain performance without compromising structural integrity. Good film cooling designs can be obtained by experienced heat transfer engineers. Nevertheless, this may be time consuming and lead to sub-optimum designs.

Film cooling design optimization is challenging because of computational cost of simulations, complexity of film cooling rows layout (which translates into a large number of design variables), and nonlinearities associated with temperature and cooling flow calculations. Also, metal temperatures represent a field, rather than a scalar, translating into a large number of responses. This paper presents a flexible framework for film cooling design optimization. Design of experiments and radial basis functions are used to reduce the computational cost. The goal attainment method is used to convert the multi objective into a constrained optimization problem. This formulation is very appealing to the designer because it does not involve the burden of analyzing complex hyper dimensional Pareto frontiers. The results obtained for a turbine blade design are an encouraging first step towards multi-disciplinary optimization aimed at achieving temperature distributions that address failure modes such as oxidation and thermal stress induced fatigue cracking.

The remaining of the paper is organized as follows. Section 2 states the optimization problem. Section 3 presents the high and low fidelity models used in this work. Section 4 presents and discusses the results. Finally, section 5 closes the paper recapitulating salient points and concluding remarks.
2 PROBLEM FORMULATION AND SOLUTION STRATEGY

Because alloys begin to melt at temperatures that are below those experienced in the operating conditions, the hot-gas-path components must be cooled. In film cooling, air is extracted from the compressor and used to cool these components. For example, the blade shown in Figure 1 is convectively cooled via serpentine passages with turbulence promoters [5].

![Figure 1: Generic example of stage 1 turbine blade [5].](image)

In this paper, the goal of the film cooling design optimization is to specify cooling hole diameters, hole to hole spacing, and film row placements so that the metal temperatures in a coated blade stay below specified targets while preserving (or saving) cooling flow. The film cooling rows are controlled by design variables relevant to diameters, row lengths, spacing between holes, radial and axial offsets for the film cooling rows.

For a given design configuration, engineers often use high-fidelity physics-based simulations to compute the field that characterizes the metal temperatures on the blade. It is virtually impossible to use such models when performing optimization (due to large number of finite element nodal temperatures). Thus, the information about temperature is further compressed in the form of zones (which, can be seen as small slices of the blade).

Ideally, one would like to minimize the temperature over the blade and the amount of used cooling flow. The large number of responses (cooling flow and maximum temperatures at multiple zones) makes multi objective strategies that generate Pareto frontiers cumbersome. On the other hand, stating the problem as a constrained optimization problem using a strategy called goal attainment (see appendix A for details) is appealing from the design point of view. After all, engineers are used to design active cooling systems to meet temperature targets while maintaining performance (translated into the amount of used cooling flow). This way, the optimization problem is defined as follow:

\[
\begin{align*}
\text{Minimize} & \quad \lambda \\
\text{such that} & \\
&t_i^{\text{max}}(x) - tol_i^T \times \lambda \leq T_i^T, \quad i = 1, 2, ..., n \\
&w(x) - tol^w \times \lambda \leq T^w \\
x^l \leq x \leq x^u, x \in \mathbb{R}^n \\
0 \leq \lambda \leq 1
\end{align*}
\]

where \( x \) is the vector of \( n \) design variables, \( t_i^{\text{max}}(x) \) are the temperature at the \( n \) zones, \( tol_i^T \) and \( T_i^T \) are temperature tolerances and targets, respectively, \( w(x) \) is the amount of cooling flow, \( tol^w \) and \( T^w \) are the cooling flow tolerance and target, respectively, and \( \lambda \) is essentially an auxiliary variable that controls how much one is willing to relax the constraints during optimization.

Here, the trade-off between temperature reduction and cooling flow is studied through two scenarios. The first one is similar to what happens in early design stages where the design is required to meet aggressive temperature targets (e.g., to reduce failure due to oxidation) with abundant cooling flow. The second scenario is when the temperature targets are relaxed so that the design meets performance requirements (saving cooling flow). Table 1 details the goal attainment setup for both scenarios. How well one can control the temperature at critical regions of the blade while saving cooling flow is also studied. This is done by assigning non-uniform temperature targets over the blade. Table 2 shows the setup for the goal attainment.

### Table 1: Scenarios used to study tradeoff between cooling flow and temperature reduction. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design). Cooling flow is also normalized between 0 (minimum within design space) and 1 (maximum within design space).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Maximum temperature</th>
<th>Cooling flow</th>
</tr>
</thead>
</table>
| 1) Aggressive maximum temperature targets with abundant cooling flow | Target: 0.7  
Target: 0.1  
(i.e., 0.7 \( \leq t_i^{\text{max}}(x) \leq 0.8 \)) | Target: 0  
Target: 1  
(i.e., 0 \( \leq w(x) \leq 1 \)) |
| 2) Relaxed maximum temperature targets with limited cooling flow | Target: 0.8  
Target: 0.2  
(i.e., 0.8 \( \leq t_i^{\text{max}}(x) \leq 1 \)) | Target: 0  
Target: 0.68  
(i.e., 0 \( \leq w(x) \leq 0.68 \)) |
Table 2: Scenarios used to study temperature targeting. Red and blue represent regions where temperature was set to be $0.9 \leq t^\text{max}(x) \leq 1$ and $0.7 \leq t^\text{max}(x) \leq 0.74$, respectively. In all three scenarios, cooling flow is set to be $0 \leq w(x) \leq 0.68$.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Temperature targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Root with low temperature targets.</td>
<td>![Red region]</td>
</tr>
<tr>
<td>2) Mid chord with low temperature targets.</td>
<td>![Blue region]</td>
</tr>
<tr>
<td>3) Mix of scenarios 1) and 2)</td>
<td>![Combined region]</td>
</tr>
</tbody>
</table>

3 COMPUTER MODELS

3.1 High-fidelity simulations

The temperature distribution on the metal surface of the turbine blade is obtained via finite element analysis (a set of in-house and commercial tools is used following GE proprietary and validated practices). Once a simulation is finished, a temperature field as shown in Figure 2 is obtained. Depending on the finite element analysis setup, the computational resources needed to perform optimization over the entire design space would be very much prohibitive. Thus, in practical terms, only a limited number of design evaluations is conducted following the guidelines of design and analysis of computer experiments.

Figure 2: Temperature field for the baseline design. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

3.2 Design of experiments and surrogate models

As mentioned, the computational cost associated with each simulation may render its use impractical for optimization. However, modern design and analysis of computer experiments tools can be used to alleviate such burden [6]-[8]. In this work, the temperature field is further compressed in the form of zones (i.e., slices of the blade). Then, surrogates for the cooling flow and the maximum temperature in each zone are built in a process involving:

1) Initial sampling of the design space: usually through space filling design of experiments [9] (such as the optimal Latin hypercube design [10]).
2) Fitting surrogates to the data: the form of the surrogate model (polynomial [11], kriging [12], radial basis function [13], etc.) is an open issue. According to the literature [14], it is difficult to know which technique works the best for a given problem and data set. Here, the guideline is fitting different forms and selecting a posteriori (e.g., based on cross validation), since this is cheap compared with the cost of simulations.
3) Refinement of the surrogate: aiming at improving prediction capabilities. Again, cross validation can be used to decide whether more points are needed. In general, the advice is that points should be added until errors fall within a limit that is tolerable (e.g., 5% relative error). If that is the case, one should add points in a space filling manner (or using engineering judgment). See [15] for further reference on sequential sampling for global metamodeling.

4 RESULTS AND DISCUSSION

4.1 Surrogate modeling

Radial basis functions (RBF) are fit to the cooling flow and maximum metal temperature. Figure 3-(a) shows the error analysis for the maximum metal temperature RBFs obtained after optimization of the RBF parameters. The fact that most the cross validation errors fall below 4% is a good indication of quality. To make sure of the good results, data were split into two sets: (i) 90% of the points to be used for fitting RBFs and (ii) 10% points for validation. Figure 3-(b) shows the results confirming that errors are expected to be below 4%.

![Cross validation analysis: average percentage cross validation error (out of all zones for the blades sampled in the design of experiment)](a)

![Validation analysis: average percentage error (out of all zones for 10% of the blades sampled in the design of experiment)](b)

Figure 3: Prediction capabilities of the surrogates for maximum temperature.
The previous approach was also applied for the surrogate of the cooling flow. Figure 4 shows that the error is expected to be very small (most of the time below 5%).

Figure 4: Validation analysis for the cooling flow surrogate: percentage error (out of a large validation set).

Figure 2 and Figure 5 illustrate the metal temperature field obtained with the high-fidelity physics-based simulations and the cheap-to-evaluate radial basis functions fitted to the data. It can be noticed that besides further spatial discretization, surrogates for maximum temperature tend to over predict the temperatures. This safely compensates for errors associated with surrogate predictions.

Figure 5: Maximum metal temperatures for the baseline design obtained with radial basis functions. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

4.2 Temperature-based optimization

Figure 6 shows the maximum temperature predictions for the designs obtained after optimization, considering scenarios shown in Table 1. The temperature targets influence the final temperature distribution. Overall, temperatures successfully fell below the assigned targets and so the cooling flow. With abundant cooling flow, a substantial portion of the blade will present temperatures below the 0.8, as seen in Figure 6-(a). Even when reducing the amount of cooling flow, it is possible to have a large portion of the blade with temperatures below 0.8, as illustrated in Figure 6-(b). Figure 7 shows the validation results (high-fidelity simulations) for each design. Both surrogates and high-fidelity simulations show the downstream effects of the film cooling rows. That is, in order to reduce temperature close to the leading edge, the mid chord ended up over cooled.

Figure 6: Temperature distribution of optimized designs (results from radial basis functions). The baseline design has 72% of the zones below 0.8 and the hottest zone is at 0.96. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

Figure 7: Actual temperature distribution for optimized designs. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

In the second scenario, shown in Table 2, certain portions of the blade have more aggressive temperature targets. This reflects design requirements such as thermal stress management. Figure 8 allows comparison between results from uniform temperature versus zone-based temperature targeting. In contrast with Figure 8-(a), where uniform temperature target was assigned, Figure 8-(b) to (c) show that the final temperature distribution is influenced by the non-uniform targets. Figure 9 shows the temperature distribution of the baseline and optimized results obtained with the high-fidelity simulations. When compared to the uniform temperature targeting of Figure 9-(a), the zone-based results of Figure 9-(b) to (d) show that cooling flow was directed to areas of high risk (lower temperature targets).
(a) Uniform temperature targets: hottest zone is at 0.89, and cooling flow is reduced by 5%.

(b) High risk assigned to radial sections 1 to 3: hottest zone is at 0.93, and cooling flow is reduced by 1%.

(c) High risk assigned to cavities sections 2 to 4: hottest zone is at 0.90, and cooling flow increases 1%.

(d) High risk assigned to radial sections 1 to 3 and cavities 2 to 4: hottest zone is at 0.92°F, and cooling flow stays the same.

Figure 8: Temperature distribution based on surrogates: baseline versus optimized designs for zone-based targeting. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

(a) Uniform temperature targets (cooling flow reduced by 5%).

(b) High risk assigned to radial sections 1 to 3 (cooling flow reduces by 1%).

(b) High risk assigned to cavities sections 2 to 4 (cooling flow increases 1%).

(c) High risk assigned to radial sections 1 to 3 and cavities 2 to 4 (cooling flow is maintained).

Figure 9: Effect of temperature targets on temperature distribution. Temperature is normalized between 0 (minimum temperature over the blade for the baseline design) and 1 (maximum temperature over the blade for the baseline design).

5 CONCLUDING REMARKS AND FUTURE RESEARCH

Cooling flow optimization is a challenging because of computational cost of the high-fidelity physics models, intricacy of film cooling row layout (which has to be defined by large number of design variables), and responses defined as nonlinear fields (as opposed to scalars). In this paper, a framework based on cheap-to-evaluate surrogate models and goal attainment is presented for the film cooling optimization based on temperature management. The approach was demonstrated for the case in which there is a tradeoff between cooling flow and temperature reduction and for the case in which temperature targets are defined according to the regions on the blade surface. Overall, it was found that:

- Temperature can be traded by cooling flow: this is very useful for preliminary design where performance and life (reliability) are being considered (once an optimal solution is achieved, robustness study can be performed to determine appropriate manufacturing tolerances for example).
- It is possible to perform optimization with zone-based temperature targeting. However, one has to consider “downstream effects” (e.g., cooling regions close to the leading edge has an impact on mid chord sections). Although targets are met from an optimization perspective, they do not reflect the final temperature field (validation using high-fidelity physics-based models should be used to predict temperature distribution).
Results for the demonstration example need to be augmented with other cases to provide a more solid estimate of the savings associated with the procedure. Nevertheless, it is safe to say that relying only on high-fidelity simulations for optimization is extremely expensive. For the demonstration example, the use of surrogates and goal attainment enabled optimization. The reduction in computational cost (with the obtained level of accuracy) is also very encouraging because it makes reliability analysis and probabilistic design optimization feasible. Manufacturing requirements, design under uncertainty, and thermal-gradients and stresses (which address failure modes such as oxidation and thermal stress induced fatigue cracking) are planned in future work.

ACKNOWLEDGEMENTS

The views expressed here reflect the views of the authors alone, and do not necessarily reflect the views of GE. Authors are very thankful to GE for supporting to publish this paper.

APPENDIX

A. GOAL ATTAINMENT: A REVIEW

The standard multi objective constrained optimization problem is:

\[
\text{Minimize } f(x) = [f_1(x) \ldots f_m(x)]
\]

such that

\[
g_j(x) \leq 0, j = 1, 2, \ldots \quad (A.2) \\
h_k(x) = 0, k = 1, 2, \ldots \\
x^l \leq x \leq x^u, x \in \mathbb{R}^n
\]

Goal attainment [16], [17] is a method for treating multi objective optimization problems. Instead of combining all objectives in a weighted sum, or creating a Pareto front, this method converts the multi objective optimization problem into a single objective constrained optimization problem. It uses a set of targets \( t = [t_1 \ldots t_m] \) associated with the set of objectives \( f(x) = [f_1(x) \ldots f_m(x)] \). Optimization will ideally make the objectives to be at least equal to the targets (i.e., \( f_i(x) \leq t_i \)). However, the formulation allows the objectives to be above the targets by a certain amount that reflects the tolerance \( tol_i \) to the violation of each constraint. The relative degree of violation is controlled by the artificially introduced single objective function and slack variable \( \lambda \). The goal attainment rewrites the optimization problem as:

\[
\text{Minimize } \lambda 
\]

such that

\[
f_i(x) - tol_i \lambda \leq t_i, i = 1, 2, \ldots, m \quad (A.4) \\
g_j(x) \leq 0, j = 1, 2, \ldots \\
h_k(x) = 0, k = 1, 2, \ldots \\
x^l \leq x \leq x^u, x \in \mathbb{R}^n \\
0 \leq \lambda \leq 1
\]

REFERENCES