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INCLUDING VIBRATION CHARACTERISTICS WITHIN COMPRESSIVE SENSING FORMULATIONS FOR STRUCTURAL MONITORING OF BEAMS

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ABSTRACT

Vibration-based monitoring of mechanical structures often involves continuous monitoring that result in high data volume and instrumentation with a large array of sensors. Previously, we have shown that Compressive Sensing (CS)-based vibration monitoring can significantly reduce both volume of data and number of sensors in temporal and spatial domains respectively. In this work, further analysis of CS-based detection and localization of structural changes is presented. Incorporating damping and noise handling in the CS algorithm improved its performance for frequency recovery. CS-based reconstruction of deflection shape of beams with fixed boundary conditions is addressed. Formulation of suitable bases with improved conditioning is explored. Restricting hyperbolic terms to lower frequencies in the basis functions improves reconstruction. An alternative is to generate an augmented basis that combines harmonic and hyperbolic terms. Incorporating known boundary conditions into the CS problem is studied.

INTRODUCTION

All types of structures and their components experience wear and tear during their operational span, which accumulate over a period of time resulting in damage and failure [1]. Early detection and localization of structural changes is thus important to prolong structural life and performance. In this respect, literature on vibration-based monitoring is elaborate and discusses several novel approaches in identifying better damage indicators [2–7]. A majority of these methods however, rely on overpopulating structures with sensors and generating a large volume of data that is arguably redundant. Therefore, our work focuses on developing an efficient vibration-based monitoring system using the undersampling technique called Compressive

Sensing (CS) [8, 9]. In our previous work, it was shown that CS-based vibration monitoring and diagnostics would enable to work with fewer data and minimal number of sensors without compromising on accuracy of detecting and localizing changes in structural characteristics [10].

Specifically, this work emphasizes on further analysis of CS-based frequency recovery and spatial response reconstruction for detection and localization of structural changes. CS was originally developed for image processing applications, where the signals and bases are very different from those required for mechanical beam vibrations. Hence, formulation of a suitable basis that better captures the characteristics of beam vibrations is important for solving the CS problem. Measurement matrix Φ , that accounts for damping is discussed using the mathematical model of a cantilever beam. In addition to this, formulating noise-handling as a part of the CS algorithm facilitated detecting shifts in frequencies from even fewer samples. Spatial response of mechanical beams sometimes contain indispensable terms in their characteristic equation that bring about numerical inconsistencies. Hyperbolic functions in the characteristic equation of a cantilever beam is a case in point. In such cases, it is important to reformulate the CS problem to handle the consequent ill-conditioning. Two approaches to handling hyperbolic functions in the basis is discussed. The first is to restrict the hyperbolic terms to lower frequencies to ensure better conditioning. The second, a more systematic approach, is to generate an augmented basis that will combine harmonic and hyperbolic terms. Effect of using prior knowledge of boundary conditions to improve localization of structural changes is also analyzed.

NOISE FORMULATION WITHIN CS

Presence of noise in measurement data is inevitable. Therefore, to evaluate practical feasibility of CS-based vibration monitoring, it is important to study its performance while handling

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noisy signal. While traditional filtering involves continuous measurement of signal, random sampling is the essence of CS. Thus, to find the sparse (CS) solution from noisy measurements, the original l_1 minimization problem (see Eq.(1)) is modified to Eq.(2), [11–13],

$$\hat{s} = \operatorname{argmin} \|s\|_1 \quad \text{s.t.} \quad \Phi s = z \quad (1)$$

$$\hat{s} = \operatorname{argmin} \|s\|_1 \quad \text{s.t.} \quad \|z - \Phi s\|_2 < \varepsilon \quad (2)$$

where ε depends on the noise variance and can be learned through experimental data. However, instead of using a quadratically constrained l_1 minimization problem, it is common practice to reformulate Eq.(2) into a LASSO problem [11] as in Eq.(3),

$$\hat{s} = \operatorname{argmin} \|\Phi s - z\|_2^2 + \lambda \|s\|_1 \quad (3)$$

where λ is a regularization parameter, dependent on the type of signal. We show the improvement provided by noise formulation in recovering natural frequencies from the impulse response of a cantilever beam. Details of the experimental setup and data acquisition are explained in our previous work, [10]. The free vibration response at a specified distance \bar{x} is given by

$$\begin{aligned} y(\bar{x}, t) &= \sum_{q=1}^{\infty} [A_q \cos(\omega_q t) + B_q \sin(\omega_q t)] W_q(\bar{x}) \\ &= \sum_{q=1}^{\infty} [\bar{A}_q(\bar{x}) \cos(\omega_q t) + \bar{B}_q(\bar{x}) \sin(\omega_q t)] \end{aligned} \quad (4)$$

where $W_q(\bar{x})$ represents the modeshape function of the beam, $\bar{A}_q(\bar{x}) = A_q W_q(\bar{x})$ and $\bar{B}_q(\bar{x}) = B_q W_q(\bar{x})$. Energy in the impulse response of the cantilever beam was concentrated in the lower modes. Therefore, to evaluate the performance of CS in the presence of noise formulation, the average l_2 error is calculated over a range of measurements $m = 20 - 100$, for recovery of the first mode (3.41Hz). The frequency range is set as $\Omega_r \in [0, \omega_r] = [0, 10]$ Hz with a resolution $\Delta\omega = 0.01$ Hz. Each mode/natural frequency has a *sine* and *cosine* component associated with it. Therefore, for recovering the first mode, the sparsity $K = 2$. Following Eq.(4), the matrix Φ is constructed using *sine* and *cosine* basis functions as given below.

$$\Phi = \begin{bmatrix} \cos(\omega_1 t_1) & \cdots & \cos(\omega_n t_1) & \sin(\omega_1 t_1) & \cdots & \sin(\omega_n t_1) \\ \cos(\omega_1 t_2) & \cdots & \cos(\omega_n t_2) & \sin(\omega_1 t_2) & \cdots & \sin(\omega_n t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cos(\omega_1 t_m) & \cdots & \cos(\omega_n t_m) & \sin(\omega_1 t_m) & \cdots & \sin(\omega_n t_m) \end{bmatrix} \quad (5)$$

For recovering only the first mode, the steady portion of the signal is used, where the higher modes and transients are negligible. Figure 1 illustrates the variation in average l_2 recovery error for varying number of measurements m . It is observed that as m increases, there is a gradual decrease in the recovery error. This may be attributed to the CS tending towards a more definitive solution. When noise formulation is incorporated in the CS problem, the recovery error is reduced for a given number of measurements. Hence, when noise is incorporated, a desired accuracy of frequency recovery may be achieved using fewer measurements.

AUGMENTED BASIS FUNCTIONS FOR FREQUENCY RECOVERY FROM DAMPED VIBRATIONS

The basis Φ , described previously in the CS problem is not a true representation of the cantilever beam system because it neglects the effect of damping. The Φ matrix was therefore augmented to account for damping in the beam and recovery error was again evaluated for $m = 20 - 100$. The damped vibration response of an n-DOF system can be expressed by Eq.(6).

$$x_i(t) = A e^{-\sigma_i t} \sin(\omega_i t + \phi) \quad \text{where, } i = 1, 2, \dots, n \quad (6)$$

where, the damping coefficient, $\sigma_i = \zeta_i \omega_i$. Thus, for recovering the first mode, the modified Φ that accounts for the effect of damping in the cantilever beam follows Eq.(6) and includes $e^{-\zeta \omega t}$. The value of ζ is calculated using *logarithmic decrement*,

$$\delta = \frac{1}{n} \log\left(\frac{P_1}{P_n}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad (7)$$

where, P_1 and P_n are the amplitudes of the first and n^{th} peaks of the impulse response respectively and n is the number of peaks/periods considered. From experiments we have, $\zeta = 0.0014$. The modified Φ that includes damping is:

$$\Phi = \begin{bmatrix} A_{11} & \cdots & A_{n1} & B_{11} & \cdots & B_{n1} \\ A_{12} & \cdots & A_{n2} & B_{12} & \cdots & B_{n2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{1m} & \cdots & A_{nm} & B_{1m} & \cdots & B_{nm} \end{bmatrix}, \quad \begin{aligned} A_{ij} &= e^{-\zeta_i \omega_i t_j} \cos(\omega_i t_j) \\ B_{ij} &= e^{-\zeta_i \omega_i t_j} \sin(\omega_i t_j) \end{aligned} \quad (8)$$

Again, considering Fig.1, it may be observed that, in addition

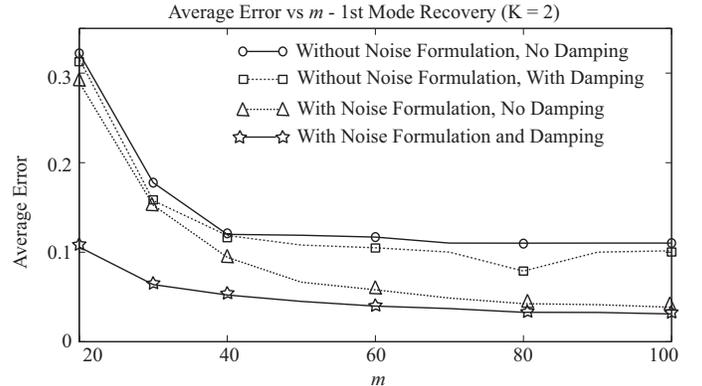


FIGURE 1: EFFECT OF NOISE FORMULATION AND DAMPING IN CS-BASED FREQUENCY RECOVERY OF CANTILEVER BEAM: AVERAGE l_2 ERROR VS m

to noise formulation in the CS-based frequency recovery, when the basis accounts for damping, for a given number of measurements, there is a significant drop in recovery error. For as low as $m = 20$, the recovery error is reduced about 67% and for $m = 100$, the error is reduced by 72%. On an average, in the presence of both noise handling as well as damping, the average recovery

error is reduced by 66%. The significance of this inquiry is especially important for reconstruction of the spatial response of a realistic beam where damping will be indispensable.

DESIGN OF MEASUREMENT MATRICES FOR SPATIAL RESPONSE RECONSTRUCTION

Localizing structural changes in a beam uses reconstruction of spatial response or operational deflection shape (ODS) that may be expressed as a weighted sum of its modeshapes, Eq.(9). When measurements are taken at random points along the beam, Φ will depend on boundary conditions. Hence, when hyperbolic components are present in the modeshape, they appear in the ODS as well as the measurement matrix. The cantilever beam is a case in point. Fundamental characteristic of mechanical beam vibrations is therefore used as the basis for formulating Φ when hyperbolic components become indispensable in the response. The cantilever beam used in this investigation is modeled using FEM simulation. The details of the FEM and CS problem setup are explained in [10]. The suitability of a measurement matrix Φ , for a given CS problem is evaluated using the Restricted Isometry Property (RIP), [11]. This mathematical aspect, that describes how well-conditioned the columns of Φ are, has been discussed in detail in [10].

$$y(x, \bar{t}) = \sum_{q=1}^{\infty} T_q(\bar{t}) W_q(x) \quad (9)$$

The magnitude of hyperbolic components $\sinh(x)$ and $\cosh(x)$ grow exponentially with increasing x . Therefore, presence of such elements in Φ make it ill-conditioned. While solving the inverse problem using l_1 minimization, such an unbounded Φ poses numerical inconsistencies and the l_1 minimization of the CS problem gives sparse solution with lower probability. Hence, it is important to devise approaches to design suitable measurement matrices for such cases. The general structure of Φ with hyperbolic terms included is,

$$\begin{bmatrix} \sin(2\pi\xi_1 x_1) & \cdots & \sin(2\pi\xi_n x_1) & \cos(2\pi\xi_1 x_1) & \cdots & \cos(2\pi\xi_n x_1) \\ \sin(2\pi\xi_1 x_2) & \cdots & \sin(2\pi\xi_n x_2) & \cos(2\pi\xi_1 x_2) & \cdots & \cos(2\pi\xi_n x_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sin(2\pi\xi_1 x_m) & \cdots & \sin(2\pi\xi_n x_m) & \cos(2\pi\xi_1 x_m) & \cdots & \cos(2\pi\xi_n x_m) \end{bmatrix}$$

$$\begin{bmatrix} \sinh(2\pi\xi_1 x_1) & \cdots & \sinh(2\pi\xi_n x_1) & \cosh(2\pi\xi_1 x_1) & \cdots & \cosh(2\pi\xi_n x_1) \\ \sinh(2\pi\xi_1 x_2) & \cdots & \sinh(2\pi\xi_n x_2) & \cosh(2\pi\xi_1 x_2) & \cdots & \cosh(2\pi\xi_n x_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sinh(2\pi\xi_1 x_m) & \cdots & \sinh(2\pi\xi_n x_m) & \cosh(2\pi\xi_1 x_m) & \cdots & \cosh(2\pi\xi_n x_m) \end{bmatrix} \quad (10)$$

To avoid numerical inconsistencies, in [14], the authors explain the use of approximate modeshapes for mechanical beams that contain hyperbolic functions. Following this, two approaches for designing Φ with hyperbolic components are examined here: (i) Uncoupled hyperbolics with restricted spatial frequency range, and (ii) Combined hyperbolic components.

1 - UNCOUPLED HYPERBOLIC COMPONENTS

CS-based reconstruction of the cantilever beam ODS is obtained from the recovered spatial frequencies and the reconstruction equation takes the form of Eq.(11) [10].

$$y(x) = a_i \sin 2\pi \xi x + b_i \cos 2\pi \xi x + c_i \sinh 2\pi \xi x + d_i \cosh 2\pi \xi x \quad (11)$$

where, a_i , b_i , c_i and d_i are the coefficients recovered using l_1 minimization. In this approach, where the hyperbolic and non-hyperbolic components are uncoupled in Φ , restricting the spatial frequency range of hyperbolic terms may be guided by the relative magnitude m_{ratio} between the hyperbolic and non-hyperbolic terms and can be obtained from Eq.(12). It is plotted in Fig.2.

$$m_1 = \sin x + \cos x, \quad m_2 = \sinh x + \cosh x, \quad m_{ratio} = m_2/m_1 \quad (12)$$

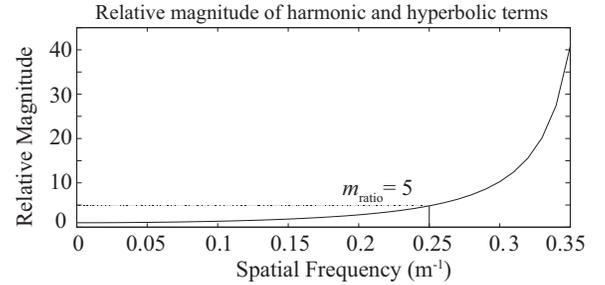


FIGURE 2: RELATIVE MAGNITUDES

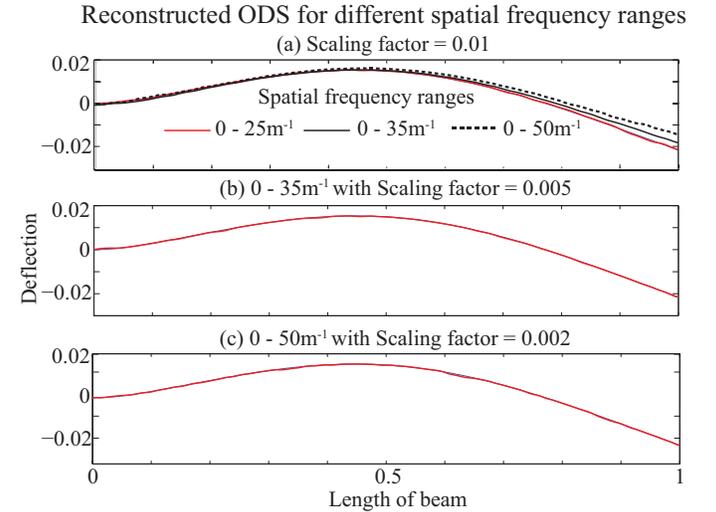


FIGURE 3: RECONSTRUCTED ODS FOR DIFFERENT CASES OF FREQUENCY RESTRICTION

Figure 2 shows the relative magnitude, m_{ratio} , as a function of increasing frequency ($0 - 0.35m^{-1}$). It can be observed that as the frequency increases, the relative magnitude increases rapidly. When the m_{ratio} is too high, numerical inconsistencies lead to failure of CS-based reconstruction. On the other hand, when m_{ratio} is too low, it eliminates the effect of hyperbolic functions from the ODS reconstruction, thus being unable to reconstruct the deflection of the free end of the cantilever beam.

A balance point of $m_{ratio} = 5$ was chosen and the hyperbolic components were scaled accordingly. The reconstruction thus obtained is illustrated in Fig.3(a) - $0 - 25m^{-1}$. The complete frequency range of operation is $0 - 25m^{-1}$ with restricted range $0 - 0.25m^{-1}$, i.e, scaling factor of 0.01. The reconstruction uses $m = 35$. Figure 3(a) also illustrates the CS-based ODS reconstruction of the cantilever beam when the complete spatial frequency ranges are $0 - 35m^{-1}$ and $0 - 50m^{-1}$ for a scaling factor of 0.01. It is observed that for these two frequency ranges, using scaling factor of 0.01 resulted in a decrease in the accuracy of reconstruction. The value of this factor therefore had to be tuned depending upon the spatial frequency range. Figure 3(b) and (c) show improved ODS reconstruction for spatial frequency ranges of $0 - 35m^{-1}$ and $0 - 50m^{-1}$. The respective scaling factor values are 0.005 and 0.002. Hence, while this approach provided a rationale for restricting the hyperbolic terms to lower frequencies, it did not help define a clear basis. An alternative approach is explored in the following section.

2 - COMBINED HYPERBOLIC COMPONENTS

This section discusses an alternative approach to designing a suitable Φ with hyperbolic terms, without restricting their frequency range. Consider the modeshape of a cantilever beam.

$$W_q(x) = [\sin \beta_q(x) - \alpha_q \cos \beta_q(x)] - [\sinh \beta_q(x) - \alpha_q \cosh \beta_q(x)] \quad (13)$$

From Eq.(13), it can be understood that for a given frequency, the hyperbolic terms reduce the magnitude of each other. In addition, for higher values of x , the magnitudes of $\sinh(x)$ and $\cosh(x)$ are almost equal to each other, thus eliminating the effect of these hyperbolic functions in the modeshape. Thus, formulating Φ , with these terms combined may help in handling the numerical inconsistencies encountered earlier with basis functions that contain hyperbolic components (Eq.(14)). Thus Φ is constructed as

$$\begin{bmatrix} \sin(2\pi\xi_1x_1) - \sinh(2\pi\xi_1x_1) - \alpha_1 \cos(2\pi\xi_1x_1) + \alpha_1 \cosh(2\pi\xi_1x_1) \cdots \\ \sin(2\pi\xi_1x_2) - \sinh(2\pi\xi_1x_2) - \alpha_1 \cos(2\pi\xi_1x_2) + \alpha_1 \cosh(2\pi\xi_1x_2) \cdots \\ \vdots \\ \sin(2\pi\xi_1x_m) - \sinh(2\pi\xi_1x_m) - \alpha_1 \cos(2\pi\xi_1x_m) + \alpha_1 \cosh(2\pi\xi_1x_m) \cdots \\ \\ \sin(2\pi\xi_nx_1) - \sinh(2\pi\xi_nx_1) - \alpha_n \cos(2\pi\xi_nx_1) + \alpha_n \cosh(2\pi\xi_nx_1) \\ \sin(2\pi\xi_nx_2) - \sinh(2\pi\xi_nx_2) - \alpha_n \cos(2\pi\xi_nx_2) + \alpha_n \cosh(2\pi\xi_nx_2) \\ \vdots \\ \sin(2\pi\xi_nx_m) - \sinh(2\pi\xi_nx_m) - \alpha_n \cos(2\pi\xi_nx_m) + \alpha_n \cosh(2\pi\xi_nx_m) \end{bmatrix} \quad (14)$$

COMPARISON OF ODS RECONSTRUCTION METHODS

This section evaluates and compares the performance of both the approaches for handling hyperbolic terms in the basis by varying the number of measurements m , and calculating the average l_2 error and probability of successful reconstruction of the cantilever ODS.

EVALUATING RECONSTRUCTION FOR UNDAMAGED CANTILEVER BEAM: The cantilever beam used for analysis in this section is undamaged, i.e. no structural changes. It is of length $L = 1$, mass density $\rho A = 1$ and rigidity modulus $EI = 1$. The FEM simulation uses $N_{el} = 1000$ elements. The cantilever beam is harmonically excited at $a = \frac{L}{5}$ with $f(t) = 5 \sin 20t$. The CS problem is setup as $\Xi_r = [\xi_l, \xi_h] = [0, 25]$, scaling = 0.01. Figure 4 (a) and (b) illustrate the variation of probability of success (PS) and average l_2 error of the cantilever beam ODS reconstruction using two different measurement matrices discussed above. An increase in the number of measurements is accompanied by decreasing reconstruction error and increasing PS . From Fig.4(b), it is observed that the average l_2 error of reconstruction using either Φ falls below 10% for a modest number of measurements.

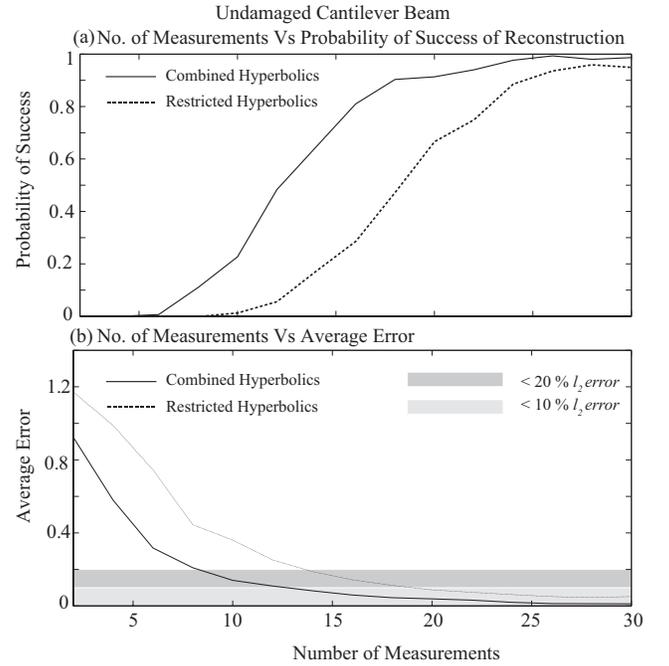


FIGURE 4: PERFORMANCE COMPARISON OF CS-BASED ODS RECONSTRUCTION METHODS

In comparison to using restricted hyperbolics, the combined hyperbolic approach exhibits better performance. While the average reconstruction error falls below 10% for $m = 15$ for Φ with combined hyperbolic terms, $m = 20$ for achieving less than 10% error for Φ with restricted hyperbolic terms. Fig.5 compares the ODS reconstruction for $m = 10$ and 20 for both types of Φ .

EVALUATING RECONSTRUCTION FOR DAMAGED CANTILEVER BEAM: The significance of ODS reconstruction lies in localization of structural change. This section therefore examines CS performance using both types of measurement matrices in the presence of structural changes in the cantilever beam. The CS-based reconstruction is evaluated under three levels of damage: (i) Low severity ($EI' = 0.9$), (ii) Medium severity ($EI' = 0.5$), (iii) High severity ($EI' = 0.1$).

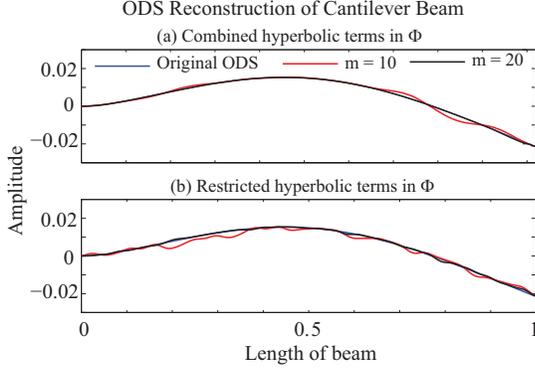


FIGURE 5: RECONSTRUCTED ODS OF CANTILEVER

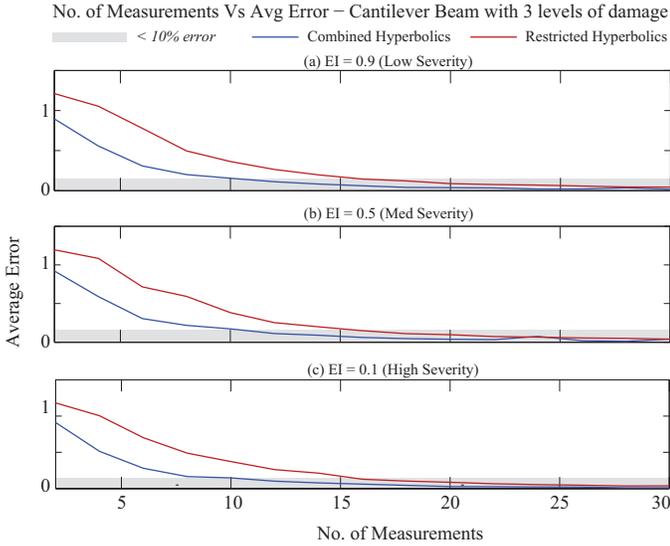


FIGURE 6: PERFORMANCE OF Φ USING RESTRICTED AND COMBINED HYPERBOLICS - DAMAGED BEAM

Figure 6 illustrates the variation of reconstruction error for increasing number of measurements for both types of Φ . Consistent with the performance results in the previous section, it is observed that the CS-based ODS reconstruction using combined hyperbolic components has lower reconstruction error for a given m when compared to restricted hyperbolic terms. CS-based ODS reconstruction of the damaged ($EI' = 0.1$) cantilever beam using both the approaches for Φ is illustrated in Fig.(7). Examining the CS-based ODS reconstruction, errors and performance results, employing a Φ where the the hyperbolic terms are combined is potentially more effective than restricting them to lower spatial frequencies. It helps to overcome the ill-conditioning of Φ while still drawing required contribution from these terms.

INCORPORATING BOUNDARY CONDITIONS IN CS

This section explores the use of boundary conditions, which are known a priori, to potentially reduce the number of data points in reconstructing the ODS of a given beam. Consider a fixed-fixed (FF) beam that is subjected to harmonic forcing $f(t) = 5 \sin 20t$ at $a = 0.2$. For the FF beam, the boundary con-

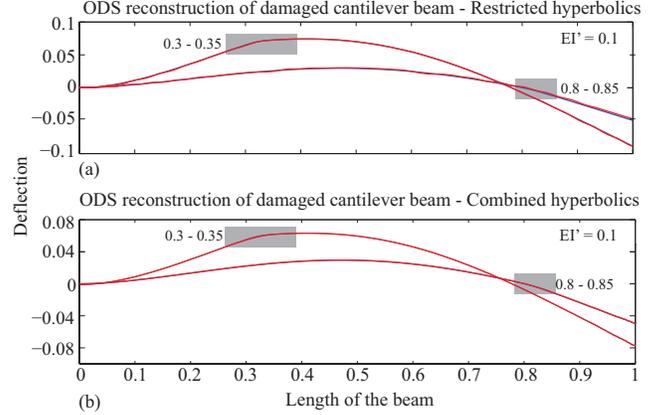


FIGURE 7: COMPARING RECONSTRUCTED ODS OF DAMAGED CANTILEVER BEAM ($EI' = 0.1$)

ditions (BCs) imply that either end of the FF beam have zero displacement and zero slope. When incorporated into the standard beam equation, the BCs give rise to a set of four equations. These are listed below:

$$W_n(x) = C_1 \sin \beta_n x + C_2 \cos \beta_n x + C_3 \sinh \beta_n x + C_4 \cosh \beta_n x \quad (15)$$

At $x = 0$ of the beam,

$$W_n(0) = C_1 \sin \beta_n 0 + C_2 \cos \beta_n 0 + C_3 \sinh \beta_n 0 + C_4 \cosh \beta_n 0 = 0 \quad (16)$$

$$W'_n(0) = D_1 \cos \beta_n 0 - D_2 \sin \beta_n 0 + D_3 \cosh \beta_n 0 + D_4 \sinh \beta_n 0 = 0 \quad (17)$$

At $x = L$ of the beam,

$$W_n(L) = C_1 \sin \beta_n L + C_2 \cos \beta_n L + C_3 \sinh \beta_n L + C_4 \cosh \beta_n L = 0 \quad (18)$$

$$W'_n(L) = D_1 \cos \beta_n L - D_2 \sin \beta_n L + D_3 \cosh \beta_n L + D_4 \sinh \beta_n L = 0 \quad (19)$$

where, $D_i = C_i \beta_n$. Recall that the CS problem is essentially the l_1 minimization of an under-sampled signal from random, linear and non-adaptive measurements. In other words, for a system represented by an under-determined set of equations, the solution to CS problem is one that has the least l_1 norm. In this context, the information from boundary conditions may be treated as four measurements in addition to those taken at random points along the length of the beam. In the spatial domain, each measurement corresponds to a sensor placed on the beam. Hence, this investigation is important because it allows for four additional measurements without the use of four more sensors. The corresponding measurement matrix Φ and measurement vector z are expressed in Eq.(20) and Eq.(21).

$$z = [0(y(0)) \quad z_1 \quad z_2 \quad \dots \quad z_m \quad 0(y(L)) \quad 0(y'(0)) \quad 0(y'(L))] \quad (20)$$

$$\Phi = \begin{bmatrix} \sin(\theta_1 x_1) & \dots & \sin(\theta_n x_1) & \cos(\theta_1 x_1) & \dots & \cos(\theta_n x_1) \\ \sin(\theta_1 x_2) & \dots & \sin(\theta_n x_2) & \cos(\theta_1 x_2) & \dots & \cos(\theta_n x_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sin(\theta_1 x_m) & \dots & \sin(\theta_n x_m) & \cos(\theta_1 x_m) & \dots & \cos(\theta_n x_m) \\ 0 & \dots & 0 & 2\pi\xi_1 & \dots & 2\pi\xi_n \\ -\theta_1 \sin(\theta_1 L) & \dots & -\theta_n \sin(\theta_n L) & \theta_1 \cos(\theta_1 L) & \dots & \theta_n \cos(\theta_n L) \end{bmatrix}, \quad (21)$$

where, $\theta_i = 2\pi\xi_i$. Figure 8 illustrates the effect of incorporating boundary conditions of the FF beam while solving the CS problem. Four cases of ODS reconstruction were examined - FF beam without structural changes and FF beam with three degrees of severity of structural change. The ODS was reconstructed for varying m and the average l_2 error was calculated over 100 trials. It is important to note that the CS problem (ODS reconstruction) with BCs always had four more measurements. From Fig.8, it can be observed that incorporating BCs did not greatly influence in bringing down the reconstruction error. When the number of measurements was lower than optimal, there was a moderate effect of incorporating the boundary conditions.

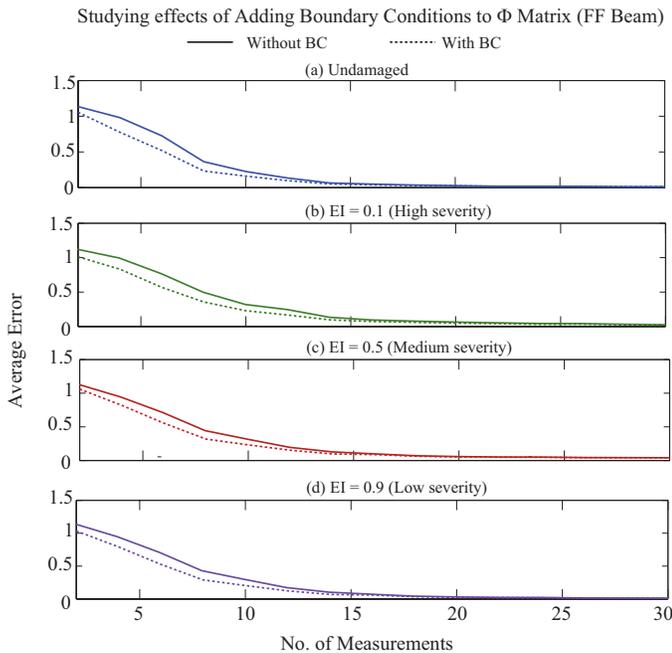


FIGURE 8: EFFECT OF INCORPORATING BOUNDARY CONDITIONS ON CS-BASED ODS RECONSTRUCTION

CONCLUSION

In continuation of our previous work, the performance of CS-based vibration monitoring was evaluated under several circumstances. Incorporating noise handling while solving the l_1 minimization problem showed an improvement in the accuracy of natural frequency recovery. In addition, further improvement in CS-based recovery was observed when the basis accounted for damping in the cantilever beam. This led to the inquiry in designing suitable measurement matrices for CS-based frequency recovery and signal reconstruction for mechanical vibrations. Specifically, in the spatial domain, two approaches to handling hyperbolic terms in Φ were discussed. Restricting hyperbolic terms to a lower range of frequencies provided an acceptable, but inconsistent method for CS-based ODS reconstruction. Combin-

ing hyperbolic and harmonic terms based on the characteristic modeshape equation of the beam showed better performance and presented an unambiguous method for the same. Further, using prior knowledge of boundary conditions while formulating the basis showed slightly improved performance of CS-based detection of structural changes at sub-optimal number of measurements.

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