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Design of Switching Laws for Shared-Sensing and Control by Reversible Transducers

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***Abstract*— In this paper we propose the concept of shared-sensing for reversible transducers. In shared-sensing, the tranducers switch between sensing and actuation modes, which is different from self-sensing where sensing and actuation are performed simultaneously. Both shared-sensing and self-sensing offer to reduce the number of transducers and provide collocation. However, our initial investigations indicate that shared-sensing has many advantages as compared to self-sensing. In this paper we develop observer based control design for systems with shared-sensing transducers. Through simulations and experimental results we demonstrate the simplicity and effectiveness with which shared-sensing and control can be implemented.**

I. Introduction

Many transducers that are widely used in control systems are reversible in nature, *i.e.*, they can be used both as sensors and actuators. Examples are electric motors, piezoelectric devices, and thermocouples. Electric motors, which are actuators, can serve as sensors through back-EMF or current sensing. The same piezoelectric device that generates strain when a voltage is applied, can generate a voltage when strain is applied. For thermocouples, Seebeck and Peltier effects define their sensing and actuation modes.

Thus far, the reversible properties of transducers have been exploited by researchers by implementing self-sensing schemes. In self-sensing, transducers are simultaneously used as sensors and actuators. This is advantageous since it leads to reduction of sensors and provides collocation. Self-sensing has been attempted in several applications but the limitations of this scheme are known and well-documented in the literature. For example, it has been shown that self-sensing magnetic bearings suffer from robustness limitations and this may explain the limited success in implementation on experimental hardware ([8] and references therein). Similarly, there are several publications that discuss the drawbacks of self-sensing in piezoelectric transducers, [2], [3], [6].

In this paper, we propose the concept of shared-sensing where sensing and actuation are not performed concurrently, but alternately. The reversible transducer switches between the roles of a sensor and an actuator and this approach has already been demonstrated for estimation and vibration cancellation in a flexible Euler-Bernoulli beam using a single piezoelectric transducer [7]. Although the system is unobservable when the transducer is used as an actuator, and uncontrollable when used as a sensor, the switched system is both controllable and observable. One of the main advantages of shared-sensing piezoelectric transducer in vibration control is that it reduces control systems hardware such as power amplifiers and data acquisition channels [7].

The goal of this paper is to explore shared-sensing with the objective of developing a general framework for observer based control design. In section II-A we investigate stabilization of open-loop-stable linear systems operating in shared-sensing mode. An example of this type of system is a flexible beam with a piezoelectric transducer and simulation results on vibration control of this system are presented in section III-A. In section II-B, we consider stabilization of open-loop-unstable linear systems. As an example, we consider the motion of an inverted pendulum about the vertical position. We demonstrate control of the inverted pendulum using an electric motor operating in shared-sensing mode. During sensing, actuation is disabled and the back-EMF is sensed to measure the angular velocity of the pendulum. There has been significant research in the area of sensorless motor control and existing techniques are based on back-EMF measurement of the silent phase, flux-linkage calculation, free-wheeling diode conduction, speed independent flux linkage function, etc., [4]. The common drawback of all these techniques is the inaccuracy of position estimation at low speeds and results in the literature indicate that current sensing can be used for velocity tracking but cannot provide satisfactory position control [1]. However, using shared-sensing we accomplish position control by balancing the inverted pendulum in the unstable vertically upright

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position. Simulation results are presented in section III-B and experimental results are provided in section IV.

II. Observer Based Control Design Using Reversible Transducers

A. Open-Loop-Stable Linear Systems

We start by considering control design for open-loop-stable systems with reversible transducers. The generic state space representation is as follows:

$$\dot{x} = Ax + Bu, \quad y = Cx \tag{1}$$

We consider shared-sensing, which implies that sensing and actuation cannot occur simultaneously but the system switches alternately between sensing and actuation modes. The switching sequence is shown below in Fig.1. δ is the time period of the switching signal, which

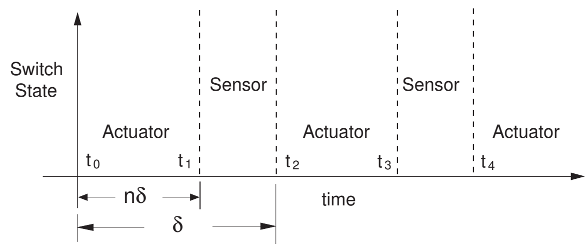

Fig. 1. Actuation and sensing sequence

consists of a fractional subinterval when the system is actuated. Let this fraction of the period δ be n. The remaining interval is used for sensing. The effect of switching action on the control input is captured as follows:

$$u = \begin{cases} u & \text{for } k\delta \leq t < (k+n)\delta \\ 0 & \text{for } (k+n)\delta \leq t < (k+1)\delta \end{cases} \quad k = 0, 1, 2, ... \tag{2}$$

Similarly, the effect on the system output is

$$y = \begin{cases} 0 & \text{for } k\delta \leq t < (k+n)\delta \\ Cx & \text{for } (k+n)\delta \leq t < (k+1)\delta \end{cases} \quad k = 0, 1, 2, ... \tag{3}$$

Thus, the system equations in the actuating mode are:

$$\begin{array}{rcl} \dot{x} & = & Ax + Bu \\ y & = & [0] \end{array} \ \forall\, k\delta \leq t < (k+n)\delta, \ k = 0, 1, 2, ... \tag{4}$$

The system equations during sensing are:

$$\begin{array}{rcl} \dot{x} & = & Ax \\ y & = & Cx \end{array} \ \forall\, (k+n)\delta \leq t < (k+1)\delta, \ k = 0, 1, 2, ... \tag{5}$$

Let us now consider the design of an observer based stabilizing control for this switched system. Denoting the observer states as $\hat{x}$ and error as $\tilde{x}$, we have the following combined system and error representation during actuation:

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}$$
$$\forall \quad k\delta \leq t < (k+n)\delta, \quad k = 0, 1, 2, ... \tag{6}$$

and the following representation during sensing:

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}$$
$$\forall \quad (k+n)\delta \leq t < (k+1)\delta, \quad k = 0, 1, 2, ... \tag{7}$$

where, $\tilde{x} = x - \hat{x}$. The combined controller based observer system therefore switches between Eqs.(6) and (7) with the former for $n\delta$ time period and the later for $(1-n)\delta$ period for every interval δ. We rewrite Eqs.(6) and (7) as follows:

$$\dot{X} = A_1 X, \quad A_1 = \begin{bmatrix} A - BK & BK \\ 0 & A \end{bmatrix},$$
$$\forall \quad k\delta \leq t < (k+n)\delta, \quad k = 0, 1, 2, ... \tag{8}$$

$$\dot{X} = A_2 X, \quad A_2 = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix},$$
$$\forall \quad (k+n)\delta \leq t < (k+1)\delta, \quad k = 0, 1, 2, ... \tag{9}$$

Note that both A_1 and A_2 are block triangular. Hence the eigenvalues of A_1 are that of $(A - BK)$ and A and those of A_2 are A and $A - LC$. We started our discussion by considering a stable open loop system. Therefore, A is Hurwitz and matrices $(A - BK)$ and $(A - LC)$ are made Hurwitz through design of K and L matrices respectively. For our analysis, instability of the switched system is avoided by ensuring that the switching interval is greater than the "*dwell time*", [5], [7].

B. Open-Loop-Unstable Linear Systems

In this section we analyze switched linear systems that are open-loop-unstable. The choice of switching interval for such systems is important. We first develop an equivalent model of such a system for small switching period δ. Consider the switching sequence shown in Fig.1. In the interval $(t_0 - t_1)$ we have the following system representation from Eqs.(8) and (9):

$$\dot{X} = A_1 X \quad \rightarrow \quad X(t_1) = e^{A_1 n\delta} X(t_0) \tag{10}$$

In the interval $(t_1 - t_2)$ we have:

$$\dot{X} = A_2 X \quad \rightarrow \quad X(t_2) = e^{A_2 (1-n)\delta} X(t_1) \tag{11}$$

Combining the two equations above yields

$$X(t_2) = e^{A_2(1-n)\delta} e^{A_1 n\delta} X(t_0) \tag{12}$$

Expanding the exponential terms in series we can show the following:

$$\lim_{\delta\to 0} e^{A_2(1-n)\delta} e^{A_1 n\delta} = e^{(A_1 n + A_2(1-n))\delta} \tag{13}$$

Hence,

$$\lim_{\delta\to 0} X(t_2) = e^{(A_1 n + A_2(1-n))\delta} X(t_0) \tag{14}$$

Similarly proceeding, we can show that

$$\lim_{\delta\to 0} X(t_3) = e^{(A_1 n + A_2(1-n))\delta} X(t_1) \tag{15}$$

Consequently, for fast switching the switched system in Eqs.(8) and (9) can be approximated by the linear system

$$\begin{aligned} \dot{X} &= [A_1 n + A_2(1-n)]\, X \\ &= \begin{bmatrix} (A - nBK) & nBK \\ 0 & (A-(1-n)LC) \end{bmatrix} X \end{aligned} \tag{16}$$

This equivalent system has the same values of the states X as the original system at discrete instants t_0, t_2, t_4, etc. and considering δ to be small, the deviation of this linear system from the original system at intermediate instants will also be small. From Eq.(16), denoting

$$A_{eq} = [A_1 n + A_2(1-n)] \tag{17}$$

we note that the eigenvalues of A_{eq} are those of $(A - nBK)$ and $(A-(1-n)LC)$. Further, the fraction n is a degree of freedom that can be exploited for faster convergence and optimality considerations.

III. Simulations

A. *Open-Loop-Stable Linear Systems*

We consider the example of a cantilever Euler-Bernoulli beam with a piezoelectric transducer operating in shared-sensing mode. A number of researchers have investigated the problem of vibration suppression in such beams and we adopt the state-space model used in [7]

$$\begin{aligned} &\dot{x} = Ax + Bu, \quad x \triangleq [\eta_1\ \eta_2\ \cdots\ \eta_k\ \dot{\eta}_1\ \dot{\eta}_2\ \cdots\ \dot{\eta}_k]^T \\ &y = Cx \end{aligned} \tag{18}$$

where,

$$A = \begin{bmatrix} \mathbf{0}_{k\times k} & \mathbf{I}_{k\times k} \\ -\mathbf{\Omega}^2 & -2\zeta\mathbf{\Omega} \end{bmatrix}, \qquad B = \begin{bmatrix} \mathbf{0}_{k\times k} \\ b_1 \\ b_2 \\ \cdots \\ b_k \end{bmatrix},$$
$$C = [c_1\ \ c_2\ \ \cdots\ \ c_k\ \ \mathbf{0}_{1\times k}] \tag{19}$$

In Eqs.(18) and (19) k is the number of modes of the beam that are being modelled, η_i represent the ith modal amplitude, $\mathbf{\Omega} = \mathrm{diag}(\omega_1, \omega_2, \cdots, \omega_k)$ represent the natural frequencies, ζ represents the structural damping, b_i and c_i are constants that are determined by the placement of the PZT sensors and actuators, respectively.

For simulation we consider a steel cantilever beam of dimensions $0.68m \times\ 0.035m \times\ 0.001m$ with one PZT as in the experimental setup in [7]. The objective is to use one PZT for controlling the beam vibrations of the first two modes. The system with the PZT was identified to have the following governing equations:

$$A = \begin{bmatrix} 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ -127.69 & 0.0 & -0.45 & 0.0 \\ 0.0 & -4225.0 & 0.0 & -2.60 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.0 \\ 0.0 \\ 2.5 \\ 10.0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.15 \\ 0.21 \\ 0.0 \\ 0.0 \end{bmatrix}^T \tag{20}$$

The system is simulated with a switching actuation and sensing sequence as given in Eqs.(2) and (3). δ and n are chosen to be 0.25s and 0.5 respectively. The matrices K and L, in Eqs.(8) and (9), are designed to place the poles of $(A-BK)$ and $(A-LC)$ at approximately -50.0 and -75.0 respectively. Initial displacements of 0.02m were applied to both modes η_1 and η_2. The simulation results are shown in Fig.2. Figs.2a, 2b, 2c, and 2d, illustrate

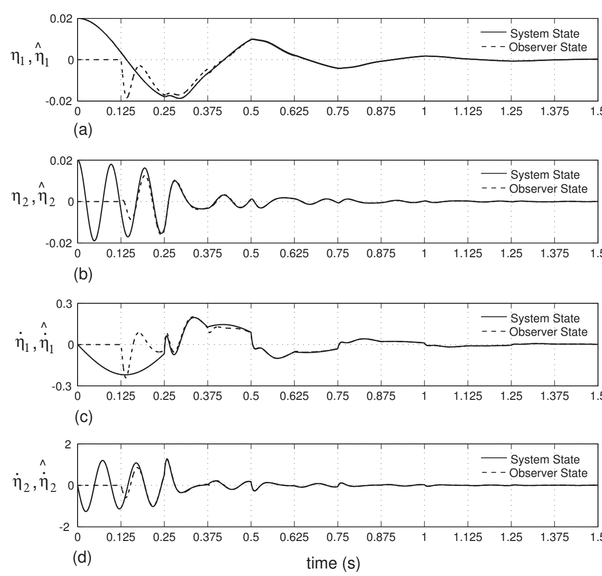

Fig. 2. Beam vibration control using reversible transducer

the evolution of the first two modal displacements and modal velocities, i.e. η_1, η_2, $\dot{\eta}_1$, and $\dot{\eta}_2$, respectively. It is noted that the system starts in actuation mode for the first 0.125s. However the observer states are initialized to zero and hence no observer dynamics is observed. From 0.125s to 0.25s the system is in sensing mode. The observer shows quick convergence during this interval. The next actuation-sensing intervals reduces oscillations significantly as shown by the plots.

B. Open-Loop-Unstable Linear Systems

We demonstrate the development in section II-B through simulation results. Consider the example of controlling an inverted pendulum mounted on a fixed base, using a DC motor as a reversible transducer. A schematic diagram of the system is shown in Fig.3. The DC motor

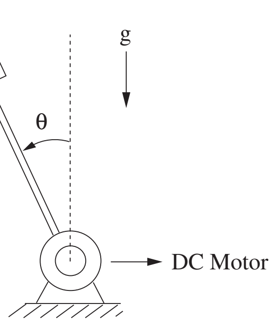

Fig. 3. Inverted pendulum on a fixed base

operates as a torque source during actuation mode and during sensing mode, the back-EMF is sensed accross the motor leads. The inverted pendulum is represented by the following linearized system equations:

$$\dot{X} = AX + Bu, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{mgL}{I} & -\frac{b}{I} & \frac{K_M}{I} \\ -\frac{K_M}{L_a} & -\frac{R_a}{L_a} & \frac{1}{L_a} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix}, \quad y = CX, \quad C = [0 \ \ K_M \ \ 0] \tag{21}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$, as shown in Fig.3, and x_3 is the motor current i. u is the supply voltage and y is the back-EMF. In the model above, m is the mass, L is the effective length, b is the damping coefficient, and I is the effective mass moment of inertia of the pendulum about the center Q. R_a, and L_a denote the internal resistance and internal inductance of the motor respectively. K_M represents the motor constant. The model parameters are derived from an experimental setup detailed in section IV. Here we list the parameter values obtained through experiments and in some cases from product manuals:

$$\begin{array}{ll} mgL = 0.0677 \ Kgm^2/s^2 & b = 0.002 \ Kgm^2/s \\ I = 8.327 \times 10^{-4} \ Kgm^2 & R_a = 9.5 \ Ohm \\ L_a = 640e \times 10^{-6} \ H & K_M = 0.047 \ Nm/amp \end{array} \tag{22}$$

The system is unstable with eigenvalues of A at 7.776, -10.460 and -14874.721. We design an observer based controller for this system. The matrix gains K and L, given in Eq.(16), are designed to stabilize A_{eq}, given in Eq.(17), by placing the eigenvalues of $(A - nBK)$ at -5, -6, and -15000, and those of $(A - (1-n)LC)$ at -25, -30, and -15000, respectively.

The simulation results are shown in Fig.4. The switching sequence in Fig.1 is applied. We show simulation results with switching period of $\delta = 0.1$s, and $n = 0.5$. The simulation results in Figs.4a, 4b, and 4c clearly

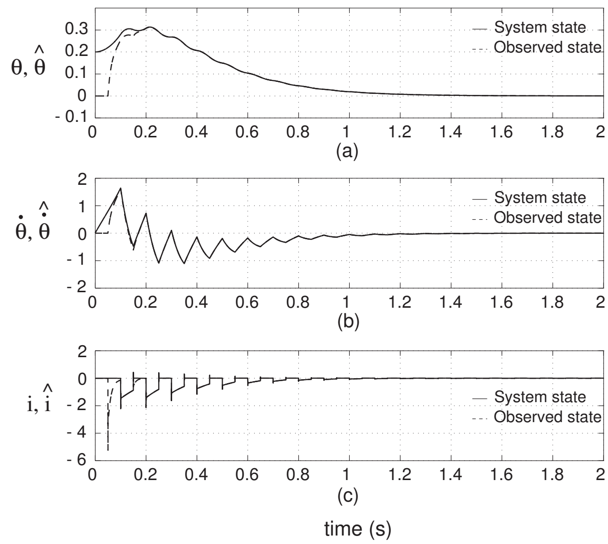

Fig. 4. Simulation results for inverted pendulum control using DC motor as reversible transducer

show the effect of actuation and sensing. The initial 0.05s is actuation interval. The actuation however is zero since the observer states are initialized to zero. During the interval from 0.05s to 0.1s the system is in sensing mode, where the estimation error rapidly diminishes. Thereafter the observed states improve further while converging the system states to the equilibrium.

IV. Experiments

Experimental results demonstrating the development of the theory in section II-A are presented in [7] and hence not included here. In this section, we experimentally demonstrate the analysis in section II-B for open-loop-unstable linear systems through the balancing of an inverted pendulum. The system parameters are given in Eq.(22). The experimental setup is shown schematically in Fig.5 and the actual hardware is shown in Fig.6. The inverted pendulum consists of a mass attached to the end of an aluminum rod that is attached to the motor shaft. The switches S_1 and S_2 are mechanical SPDT relays that are switched using 10V logic signals. Both the relays are switched simultaneously to either NO (Normally Open) or NC (Normally Closed) position. When the relays are in the NO position, the motor is powered on and the system is in the actuation mode. The motor is powered using a switched power amplifier. In the NC position, the power to the motor is disconnected and the back-EMF is sensed. The 10V logic signals for switching the relays as well as the observer based controller are programmed in $MATLAB^{®}/Simulink^{®}$ and executed in real-time using a $dSPACE^{®}$ $DS1104$ controller.

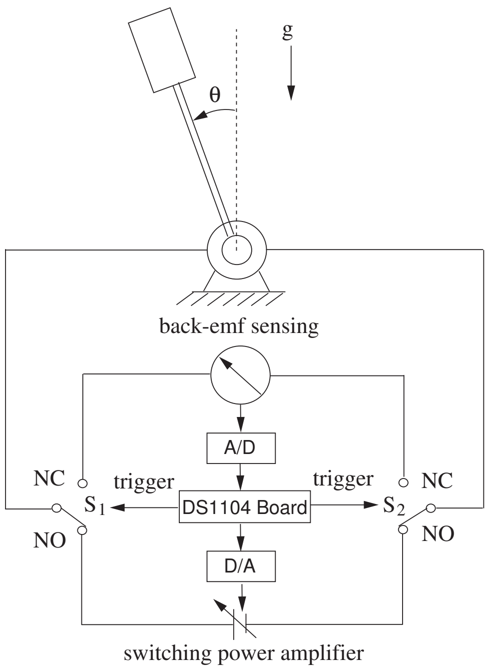

Fig. 5. Shared-sensing and actuation setup for inverted pendulum

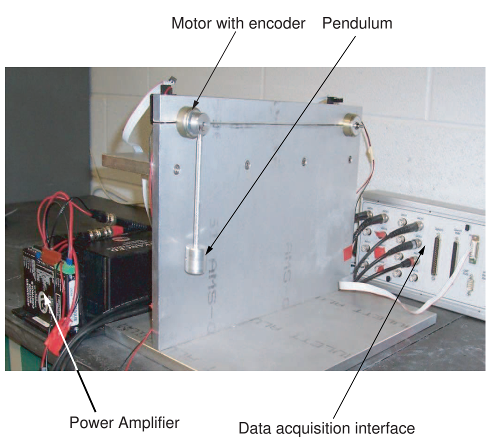

Fig. 6. Experimental test stand

For the experiment, $\delta = 0.1$s and $n = 0.5$ as in the simulation. The actuation and sensing pulses are shown in Fig.7. The switches S_1 and S_2 are triggered with the

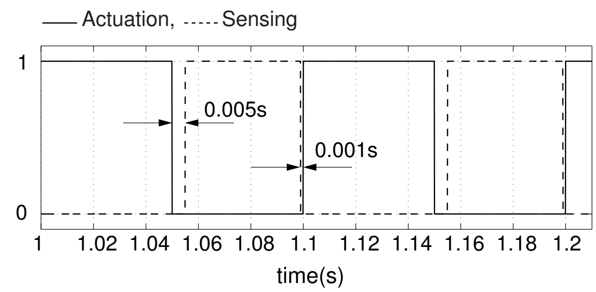

Fig. 7. Actuation and sensing pulses for the experimental setup

actuation pulse in Fig.7. The sensing pulse, shown in Fig.7, determines when the observer is active. Note that the sensing pulse starts after a lag of 0.005s and ends 0.001s before the actuation is initiated. This prevents the voltage pulses associated with the switching action of the relays from affecting the back-EMF measurement.

The angle of the pendulum relative to the vertically downward position was measured by an encoder. This angle α, and the estimated angular position of the pendulum $\hat{\theta}$ (measured relative to the vertically upright position), are shown in Fig.8a. In the experiment, the pendulum was initially in the vertically downward stable equilibrium position. The pendulum was manually turned during the interval t_1 to t_2, to approximately 160^{o}. At t_2, the observer based control was invoked and the pendulum stablized in the inverted position. For the observer based control, the gain matrices K and L, given in Eq.(16) are designed to stabilize A_{eq}, given in Eq.(17), by placing the eigenvalues of $(A - nBK)$ at -5, -6, and -10000, and those of $(A - (1-n)LC)$ at -25, -30, and -10000, respectively. At instants t_3, t_4, and t_5, the position of the pendulum was manually perturbed in either direction by upto 20^{o} and the shared-sensing based control was shown to stabilize the system. In Figs.8b, 8c, and 8d we show the measured velocity, applied voltage, and the motor current, respectively. Note that the sudden changes in the motor voltage, current, and speed, at instants t_3, t_4, and t_5 are due to the manual perturbations applied on the pendulum, and that at t_2 is due to the initiation of control which stabilizes the pendulum. In Fig.8b, the change in the measured velocity of the pendulum during the interval t_1 to t_2 is attributed to the manual turning of the pendulum from the downward vertical position to about 20^{o} from the upward vertical. The saturation in the applied voltage in Fig.8d is due to the ± 10V range for analog output channels.

The switching of commanded voltage and the effect of switching on the observed position are shown in detail in Figs.9b and 9c respectively. Clearly, the applied voltage

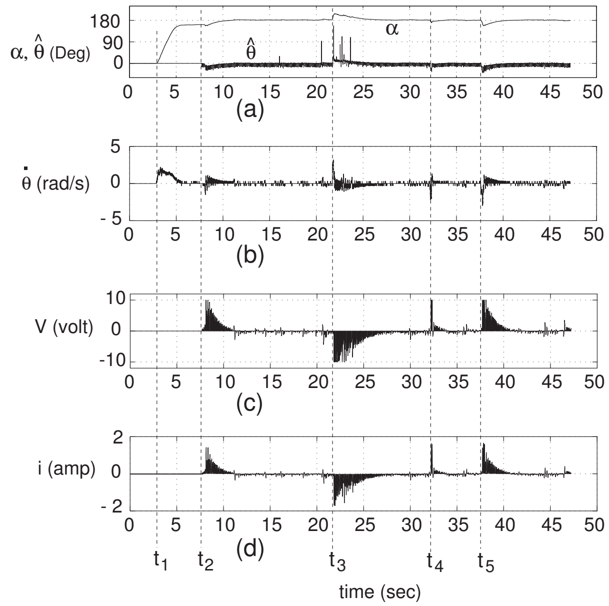

Fig. 8. Pendulum measured and observed variables

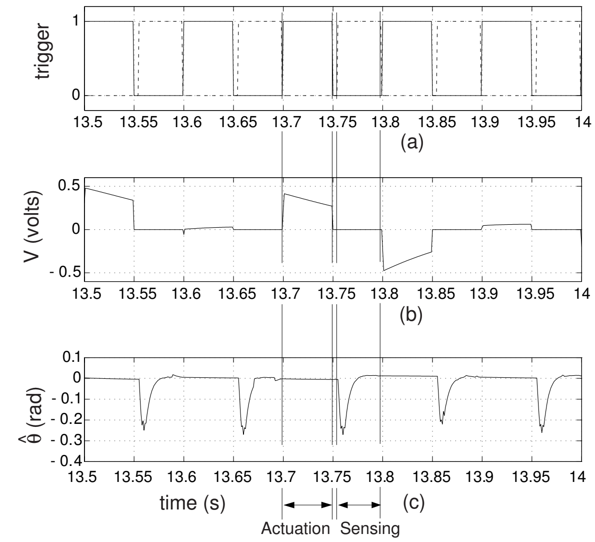

Fig. 9. Variation of motor voltage command and observed pendulum position with switching

on the motor is zero during the sensing intervals and nonzero otherwise. Also note that the observer error in the position increases sharply as soon as the transition to sensing mode occurs. Thereafter, the observation improves rapidly. Since the actuation is zero during sensing, this momentary large observation error does not have any effect on the applied voltage.

V. Conclusion

In this paper we propose the shared-sensing approach towards control of systems with reversible transducers. In contrast to self-sensing, where sensing and actuation are simultaneously performed, shared-sensing is realized by switching the reversible transducers between sensing and actuation modes. We design observer based controllers for two categories of systems, namely, open-loop-stable and open-loop-unstable linear systems. For the former, we demonstrate shared-sensing through vibration suppression of an Euler-Bernoulli beam using piezoelectric transducers. For this system, multiple researchers have indicated the drawbacks of self-sensing. For the later, we demonstrate shared-sensing through balancing of an inverted pendulum using an electric motor. Here, it should be mentioned that position control of motors using self-sensing has been investigated by many researchers but with limited success. Our results are encouraging and indicate that shared-sensing holds considerable promise and deserves further investigation.

VI. Acknowledgement

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References

[1] M. Bodson and J. Chiasson. A comparison of sensorless speed estimation methods for induction motor control. *Proc. American Control Conference, Anchorage, AK*, pages 3076–3081, 2002.

[2] S. Carabelli and A. Tonoli. System propoerties of flexible stuctures with self-sensing piezoelectric transducers. *Journal of Sound and Vibration*, 235(1):1–23, 2000.

[3] J. Holterman and T. J. A. de Vries. Active and passive damping based on piezoelectric elements - controllability issues. *Proc. 3rd Workshop on European Scientific and Industrial Collaboration, Enschede, The Netherlands*, pages 179–188, 2001.

[4] T. Kim and M. Ehsani. Sensorless control of the bldc motors from near-zero to high speeds. *IEEE Transactions on Power Electronics*, 19(6):1635–1645, 2004.

[5] D. Liberzon and A. S. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 19(5):59–70, 1999.

[6] S. O. Reza Moheimani. A survey of recent innovations in vibration damping and control using shunted piezoelectric transducers. *IEEE Trans. Control System Technology*, 11(4):482–494, 2003.

[7] S. S. Nudehi and R. Mukherjee. Enhancing controllability and observability in underactuated and undersensed systems through switching: Application to vibration control. *ASME Journal of Dynamic Systems, Measurement, and Control*, 126:790–796, 2004.

[8] N. M. Thibeault and R. S. Smith. Magnetic bearing measurement configurations and associated robustness and performance limiations. *ASME Journal of Dynamic Systems, Measurement, and Control*, 124:589–598, 2002.