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An Impulse-Momentum Approach to Swing-Up Control of the Pendubot

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***Abstract*— The standard control problem of the pendubot refers to the task of stabilizing its equilibrium configuration with the highest potential energy. Linearization of the dynamics of the pendubot about this equilibrium results in a completely controllable system and allows a linear controller to be designed for local asymptotic stability. For the under-actuated system, the important task is therefore to design a controller that will swing up both links and bring the configuration variables of the system within the region of attraction of the desired equilibrium. This paper provides a method for swing-up control based on a series of rest-to-rest maneuvers of the first link about its vertically upright configuration. The rest-to-rest maneuvers are designed such that each maneuver results in a net gain in energy of the second link. This results in swing-up of the second link and the pendubot configuration reaching the region of attraction of the desired equilibrium. A four-step algorithm is provided for swing-up control followed by stabilization. Simulation results are presented to demonstrate the efficacy of the approach.**

NOMENCLATURE

For the nomenclature below, the subscript i assumes values $1, 2$, and j assumes values $1, 2, 3, 4, 5$.

l_i	length of the i-th link, (m)
d_i	distance between the i-th joint and center of mass of the i-th link, (m)
m_i	mass of the i-th link, (kg)
I_i	mass moment of inertia of the i-th link about its center of mass, (kgm^2)
θ_i	angular displacement of the i-th link as shown in Fig.1, (rad)
$\dot{\theta}_i$	angular velocity of the i-th link, (rad/s)
$\dot{\theta}_i^-$	angular velocity of the i-th link, immediately before the first link is stopped, (rad/s)
$\dot{\theta}_2^+$	angular velocity of the second link, immediately after the first link is stopped, (rad/s)
v_2	velocity of the center of mass of the second link, (m/s)
v_2^-	velocity of the center of mass of the second link, immediately before the first link is stopped, (m/s)
v_2^+	velocity of the center of mass of the second link, immediately after the first link is stopped, (m/s)
XY	inertial reference frame with unit vectors $\vec{i}$ and $\vec{j}$ along the X and Y axes, respectively
xy	Cartesian reference frame fixed to the second link
F_x	force acting on the second link at the second joint along the x direction, (N)
F_y	force acting on the second link at the second joint along the y direction, (N)
F_{imp}	impulsive force acting on the second link at the second joint, (N)
F	force acting on the second link at the second joint along the direction of motion of the second joint (N)
τ	external torque applied on the first link, (Nm)
τ_h	external torque required to hold the first joint fixed, *i.e.*, maintain $\dot{\theta}_1 = 0$, (Nm)
τ_b	external torque required for braking, *i.e.*, causing exponential decay in the velocity of the first link, (Nm)
M_{imp}	impulsive moment acting on the second link at its center of mass, (Nm)
E_2	total energy of the second link, (J)
E_{2T}	potential energy of the second link when $(\theta_1, \theta_2) = (\pi/2, 0)$, (J)
g	acceleration due to gravity, $(9.81\ m/s^2)$
q_j	constants, who values depend on kinematic and dynamics parameters of the pendubot
S_i	$\sin\theta_i$
C_i	$\cos\theta_i$
S_{12}	$\sin(\theta_1 + \theta_2)$
C_{12}	$\cos(\theta_1 + \theta_2)$

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I. INTRODUCTION

The Pendubot [2, **?**] is a two-link robot in the vertical plane with an actuator at the shoulder joint but no actuator at the elbow joint. The control problem of the pendubot typically refers to the task of stabilizing its equilibrium configuration with the highest potential energy, which is unstable. It is a classic example of an underactuated system [10] and its control problem has similarities with the control problems of the single and double inverted pendulums on a cart, the planar underactuated robot [1], and the Acrobot [3].

The complete control of the pendubot requires swing-up control that brings the configuration of the pendubot close to its equilibrium configuration with the highest potential energy followed by balance control that stabilizes the desired equilibrium. The balance control problem was addressed by several researchers. For example, Spong and Block [9] linearized the dynamic equations and used linear quadratic

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regulator theory for pole placement and Zhang and Tarn [11] used hybrid control methods. In this paper we linearize the dynamic equations and use a linear controller for balance control like Spong and Block [9], but our main contribution lies in the development of a new method for swing-up control of the pendubot.

For the swing-up control problem, Spong and Block [9] proposed a method based on feedback linearization and Fantoni, et al. [4] developed an energy-based control method based on stability analysis. Grognard and Canudas-de-Wit [7] and Orlov, et al. [8] proposed methods based on zero dynamics of the system that result in limit cycle behavior, and more recently, Freidovich, et al. [5] presented a control methodology based on imposition of virtual holonomic constraints. Our approach to swing-up control is similar to the energy based approach of Fantoni, et al. [4] but we focus on the force interaction between the two links and the change in energy of the second link resulting from it. Our method is based on a series of rest-to-rest maneuvers of the first link about its vertically upright configuration that results in swinging-up of the second link and the pendubot configuration reaching the region of attraction of the desired equilibrium point. The linear controller is invoked at this juncture to stabilize the equilibrium.

The rest-to-rest maneuvers proposed in our approach require impulsive moments to be applied to the actuated joint of the pendubot. Such impulsive forces/moments can be used for control of the acrobot, biped walking robots, and underactuated dynamical systems in general. Although we do not discuss these problems in this paper, it should be mentioned that impulsive forces and moments are particularly useful for control of underactuated systems with joint limits such as the biped walking robot where the torso has to remain close to vertically upright at all times. For the pendubot, where the first link can be easily taken to the vertically upright configuration, the region of attraction of the desired equilibrium can be reached by increasing the energy of the second link while maintaining the first link in the neighborhood of its vertically upright configuration. This approach to swing-up control also poses limits on the motion of the first link.

This paper is organized as follows. In section 2 we provide the equations of motion of the pendubot and derive expressions for the force of the interaction between the two links, the holding torque, and the braking torque. In section 3 we design rest-to-rest maneuvers of the first link about its vertically upright configuration that result in net gain in energy of the second link. It is assumed that the first link is brought to rest instantaneously at the end of each maneuver and this can be modeled by an impulsive force and an impulsive moment acting on the second link. The algorithm for swing-up control followed by stabilization of the desired equilibrium is discussed in section 4. Section 5 provides simulation results that validate the impulse-momentum model of rapid braking and demonstrate swing-up control of the pendubot. Concluding remarks are provided in section 6.

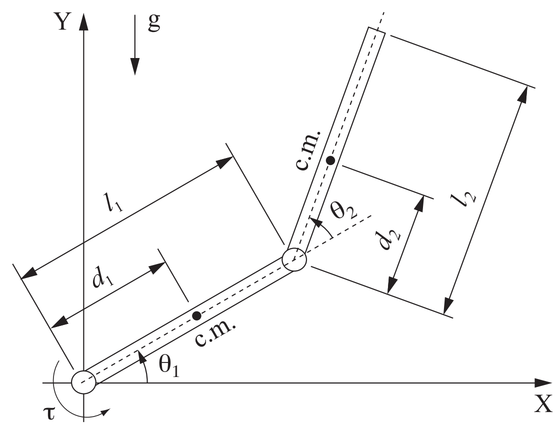

Fig. 1. The pendubot

II. System Dynamics

A. Equations of motion

Consider the pendubot in Fig.1. Assuming no friction in the joints, the equation of motion can be obtained using the Lagrangian formulation as follows [4]

$$A(\theta)\ddot{\theta} + B(\theta,\dot{\theta})\dot{\theta} + G(\theta) = T \tag{1}$$

where

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \qquad T = \begin{pmatrix} \tau \\ 0 \end{pmatrix} \tag{2}$$

and $A(\theta)$, $B(\theta,\dot{\theta})$, and $G(\theta)$, given by the expressions

$$A(\theta) = \begin{bmatrix} q_1 + q_2 + 2q_3C_2 & q_2 + q_3C_2 \\ q_2 + q_3C_2 & q_2 \end{bmatrix} \tag{3}$$

$$B(\theta,\dot{\theta}) = q_3S_2 \begin{bmatrix} -\dot{\theta}_2 & -(\dot{\theta}_1 + \dot{\theta}_2) \\ \dot{\theta}_1 & 0 \end{bmatrix} \tag{4}$$

$$G(\theta) = g \begin{bmatrix} q_4C_1 + q_5C_{12} \\ q_5C_{12} \end{bmatrix} \tag{5}$$

are the inertia matrix, matrix containing terms resulting in the Coriolis and centrifugal forces, and vector of gravity forces, respectively. In Eqs.(3), (4), and (5), q_i, $i = 1, 2, \cdots, 5$ are constants and have the following expressions [4]

$$\begin{aligned} q_1 &= m_1d_1^2 + m_2l_1^2 + I_1 \\ q_2 &= m_2d_2^2 + I_2 \\ q_3 &= m_2l_1d_2 \\ q_4 &= m_1d_1 + m_2l_1 \\ q_5 &= m_2d_2 \end{aligned} \tag{6}$$

B. Force of interaction between the two links

By applying the Newton-Euler method [6], the force of interaction between the two links can be computed as follows

$$\begin{aligned} F_x &= m_2\left[-d_2(\dot{\theta}_1 + \dot{\theta}_2)^2 + l_1(\ddot{\theta}_1S_2 - \dot{\theta}_1^2C_2) + gS_{12}\right] \\ F_y &= m_2\left[d_2(\ddot{\theta}_1 + \ddot{\theta}_2) + l_1(\ddot{\theta}_1C_2 + \dot{\theta}_1^2S_2) + gC_{12}\right] \end{aligned} \tag{7}$$

The forces F_x and F_y act along the x and y directions, respectively, as shown in Fig.2. The resultant of F_x and

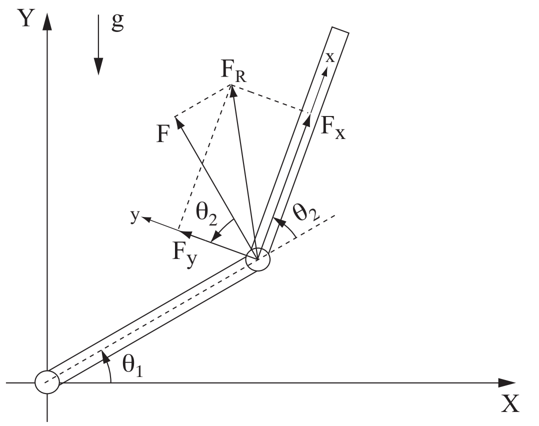

Fig. 2. Force of interaction between the two links

F_y, denoted by F_R, can be decomposed into a workless constraint force along the length of the first link and the component F that does positive work on the second link. F can be expressed in terms of F_x and F_y as follows

$$\begin{aligned} F &= F_x\,S_2 + F_y\,C_2 \\ &= m_2\Big[l_1\ddot{\theta}_1 + d_2(\ddot{\theta}_1+\ddot{\theta}_2)C_2 \\ &\qquad -d_2(\dot{\theta}_1+\dot{\theta}_2)^2 S_2 + gC_1\Big] \end{aligned} \tag{8}$$

The total energy of the second link can be expressed as follows

$$E_2 = \frac{1}{2}I_2(\dot{\theta}_1+\dot{\theta}_2)^2 + \frac{1}{2}m_2\vec{v}_2\cdot\vec{v}_2 + m_2\,g(l_1S_1+d_2S_{12}) \tag{9}$$

where $\vec{v}_2$ is given by the expression

$$\begin{aligned} \vec{v}_2 &= -\Big[l_1\,\dot{\theta}_1S_1 + d_2(\dot{\theta}_1+\dot{\theta}_2)S_{12}\Big]\,\vec{i} \\ &\quad +\Big[l_1\,\dot{\theta}_1C_1 + d_2(\dot{\theta}_1+\dot{\theta}_2)C_{12}\Big]\,\vec{j} \end{aligned} \tag{10}$$

By differentiating the expression for E_2, we get

$$\begin{aligned} \dot{E}_2 &= m_2l_1\dot{\theta}_1\Big[l_1\ddot{\theta}_1 + d_2(\ddot{\theta}_1+\ddot{\theta}_2)C_2 \\ &\qquad -d_2(\dot{\theta}_1+\dot{\theta}_2)^2S_2 + gC_1\Big] \end{aligned} \tag{11}$$

It can be verified from Eq.(8) that $\dot{E}_2 = F\,l_1\dot{\theta}_1$. This is not surprising since $l_1\dot{\theta}_1$ is the velocity of the point of application of the force F and has the same direction as that of F.

C. Holding torque

We compute the torque required to hold the first link fixed, *i.e.*, maintain $\dot{\theta}_1 = 0$. By substituting $\dot{\theta}_1 = \ddot{\theta}_1 = 0$ in Eq.(1), we get

$$\begin{bmatrix} q_2+q_3C_2 \\ q_2 \end{bmatrix}\ddot{\theta}_2 - \begin{bmatrix} \dot{\theta}_1+\dot{\theta}_2 \\ 0 \end{bmatrix} q_3S_2\,\dot{\theta}_2 + g\begin{bmatrix} q_4C_1+q_5C_{12} \\ q_5C_{12} \end{bmatrix} = \begin{pmatrix} \tau_h \\ 0 \end{pmatrix} \tag{12}$$

By eliminating $\ddot{\theta}_2$ from the two equations in Eq.(12), τ_h can be expressed as follows

$$\tau_h = -q_3S_2\,\dot{\theta}_2^2 + g\left[q_4C_1 - \frac{q_3q_5}{q_2}C_{12}\right] \tag{13}$$

The holding torque in Eq.(13) will be used in section 4 in our algorithm for swing-up control.

D. Braking torque

We consider braking action that results in exponential decay of $\dot{\theta}_1$ to zero. Therefore, we assume

$$\ddot{\theta}_1 = -k_1\dot{\theta}_1, \quad k_1 > 0 \tag{14}$$

where k_1 is a constant that will control the rate of decay of $\dot{\theta}_1$. To compute the torque required for braking, we multiply Eq.(1) with the inverse of the inertia matrix to obtain

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \frac{1}{q_1q_2 - q_3^2C_2^2}\begin{bmatrix} q_2\tau + h_1 \\ -(q_2+q_3C_2)\tau + h_2 \end{bmatrix} \tag{15}$$

where h_1 and h_2 are given by the expressions

$$\begin{aligned} h_1 &= q_2q_3(\dot{\theta}_1+\dot{\theta}_2)^2S_2 + q_3^2\dot{\theta}_1^2S_2C_2 \\ &\quad +g(q_3q_5C_2C_{12} - q_2q_4C_1) \end{aligned} \tag{16}$$

$$\begin{aligned} h_2 &= -(\dot{\theta}_1+\dot{\theta}_2)^2(q_2q_3+q_3^2C_2)S_2 \\ &\quad -(q_1+q_3C_2)q_3\dot{\theta}_1^2S_2 - g\,\{q_3q_5C_2C_{12} \\ &\qquad -(q_2+q_3C_2)q_4C_1 + q_1q_5C_{12}\} \end{aligned} \tag{17}$$

On substituting Eq.(14) in the first equation of Eq.(15), we get

$$\tau_b = -\frac{1}{q_2}\left[k_1\dot{\theta}_1(q_1q_2 - q_3^2C_2^2) + h_1\right] \tag{18}$$

When the first joint comes to rest, the braking torque becomes equal to the holding torque. This can be verified from Eqs.(13) and (18).

III. Energy Considerations of the Second Link

A. Effect of suddenly stopping the first link

A large value of gain k_1 in the expression for the braking torque in Eq.(18) will result in sudden stopping of the first link. This action of suddenly stopping the first link has the effect of an impulsive force and an impulsive moment being applied on the second link, as shown in Fig.3. The impulsive force results in a change in linear momentum of the second link and the impulsive moment results in a change in angular momentum. The change in momenta can be expressed as follows

$$\vec{F}_{imp}\,\Delta t = m_2\,(\vec{v_2^+} - \vec{v_2^-}) \tag{19}$$

$$\vec{M}_{imp}\,\Delta t = \vec{r_2}\times\vec{F}_{imp}\,\Delta t = I_2\,\dot{\theta}_2^+ - I_2\,(\dot{\theta}_1^- + \dot{\theta}_2^-) \tag{20}$$

where Δt is the short interval of time over which the impulsive force and impulsive moment act, and $\vec{v_2^+}$ and $\vec{v_2^-}$

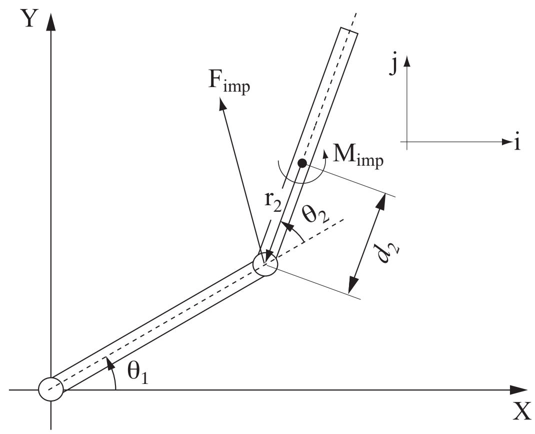

Fig. 3. Effect of suddenly stopping the first link

are given by the expressions

$$\vec{v_2^+} = d_2\,\dot\theta_2^+\,(-S_{12}\vec{i}+C_{12}\vec{j}) \tag{21}$$

$$\begin{aligned}\vec{v_2^-} &= -\left[l_1\,\dot\theta_1^- S_1 + d_2(\dot\theta_1^- + \dot\theta_2^-)S_{12}\right]\vec{i} \\ &\quad + \left[l_1\,\dot\theta_1^- C_1 + d_2(\dot\theta_1^- + \dot\theta_2^-)C_{12}\right]\vec{j}\end{aligned} \tag{22}$$

that can be obtained from Eq.(10). The vector $\vec{r_2}$ is shown in Fig.3 and has the form

$$\vec{r_2} = -d_2(C_{12}\vec{i} + S_{12}\vec{j}) \tag{23}$$

By substituting Eqs.(19), (21), (22) and (23) into Eq.(20), we get

$$\dot\theta_2^+ = \dot\theta_2^- + \left[1 + \frac{l_1 m_2 d_2 C_2}{I_2 + m_2 d_2^2}\right]\dot\theta_1^- \tag{24}$$

Since there is no change in potential energy of the second link over the Δt time interval, the change in total energy of the second link is due to the change in its kinetic energy alone, and is equal to

$$\begin{aligned}\Delta E_2 &= \frac{1}{2}(I_2 + m_2 d_2^2)(\dot\theta_2^+)^2 - \frac{1}{2}I_2\left(\dot\theta_1^- + \dot\theta_2^-\right)^2 \\ &\quad -\frac{1}{2}m_2\,\left[l_1^2\,(\dot\theta_1^-)^2 + d_2^2(\dot\theta_1^- + \dot\theta_2^-)^2\right. \\ &\quad \left. +2l_1 d_2\dot\theta_1^-(\dot\theta_1^- + \dot\theta_2^-)C_2\right]\end{aligned} \tag{25}$$

By substituting Eq.(24) into Eq.(25), we can express the change in the total energy of the second link in terms of the velocity of the first link prior to stopping, as follows

$$\Delta E_2 = \frac{1}{2}m_2 l_1^2\left[\frac{m_2 d_2^2 C_2^2}{I_2 + m_2 d_2^2} - 1\right](\dot\theta_1^-)^2 \tag{26}$$

Since $m_2 d_2^2 C_2^2 < (I_2 + m_2 d_2^2)$, $\Delta E_2 \le 0$ and $\Delta E_2 = 0$ if and only if $\dot\theta_1^- = 0$. Clearly, the total energy of the second link decreases whenever the first link is stopped suddenly.

B. Rest-to-rest maneuver of the first link

Consider a maneuver in which the first joint starts from rest and is brought back to rest through the application of a braking torque using a large gain k_1. Taking into account the loss of energy due to sudden stopping, given by Eq.(26), the net work done on the second link due to the rest-to-rest maneuver can be computed as follows

$$\begin{aligned}\Delta E_2 &= \int F\,l_1 d\theta_1 + \frac{1}{2}m_2 l_1^2\left[\frac{m_2 d_2^2 C_2^2}{I_2 + m_2 d_2^2} - 1\right](\dot\theta_1^-)^2 \\ &\ge \int F\,l_1\dot\theta_1 dt - \frac{1}{2}m_2 l_1^2(\dot\theta_1^-)^2\end{aligned} \tag{27}$$

where F is given by the expression in Eq.(8). If we now impose the constraint

$$d_2(\ddot\theta_1 + \ddot\theta_2)C_2 - d_2(\dot\theta_1 + \dot\theta_2)^2 S_2 + gC_1 = k_2\,l_1\ddot\theta_1, \quad k_2 > 0 \tag{28}$$

we get from Eqs.(8) and (27)

$$\begin{aligned}\Delta E_2 &\ge l_1\int F\,\dot\theta_1 dt - \frac{1}{2}m_2 l_1^2(\dot\theta_1^-)^2 \\ &= l_1\int m_2\left[(1+k_2)l_1\,\ddot\theta_1\right]\dot\theta_1 dt - \frac{1}{2}m_2 l_1^2(\dot\theta_1^-)^2 \\ &= \left[\frac{1+k_2}{2}\right]m_2\,l_1^2\int 2\dot\theta_1\ddot\theta_1\,dt - \frac{1}{2}m_2 l_1^2(\dot\theta_1^-)^2 \\ &= \left[\frac{1+k_2}{2}\right]m_2\,l_1^2(\dot\theta_1^-)^2 - \frac{1}{2}m_2 l_1^2(\dot\theta_1^-)^2 \\ &= \frac{1}{2}k_2\,m_2\,l_1^2(\dot\theta_1^-)^2 > 0\end{aligned} \tag{29}$$

Using Eq.(1) it can be shown that the constraint in Eq.(28) can be imposed by applying the torque

$$\begin{aligned}\tau &= \left[\frac{q_1 q_2 - q_3^2 C_2^2}{k_2 l_1 q_2 + d_2 q_3 C_2^2}\right]\left\{gC_1 - d_2(\dot\theta_1 + \dot\theta_2)^2 S_2\right\} \\ &\quad + \left[\frac{1}{k_2 l_1 q_2 + d_2 q_3 C_2^2}\right]\{d_2(h_1 + h_2)C_2 - k_2 l_1 h_1\}\end{aligned} \tag{30}$$

where h_1 and h_2 were defined earlier by Eqs.(16) and (17). Clearly, the net energy of the second link will increase if the first joint is driven using the torque expression in Eq.(30) and then stopped suddenly.

IV. Algorithm for Swing-Up Control

A four-step algorithm is proposed for swing-up control of the pendubot followed by asymptotic stabilization of the desired equilibrium.

1. Initialization:

- Linearize the dynamic equations of the pendubot in Eq.(1) about the desired equilibrium $(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (\pi/2, 0, 0, 0)$.
- Using the model of the linearized system, design a linear controller to render the desired equilibrium of the pendubot (nonlinear system) locally asymptotically stable.
- Choose a small angle α, $\alpha > 0$, such that $(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (\theta_1, 0, 0, 0)$ lies in the region of

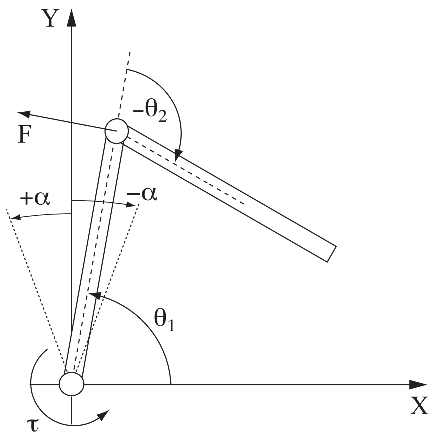

Fig. 4. Swing-up control of the first link

attraction of the desired equilibrium $(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (\pi/2, 0, 0, 0)$ for all values of θ_1 satisfying $(\pi/2-\alpha) \leq \theta_1 \leq (\pi/2+\alpha)$.

2. Swing-up control of the first link:

Drive the first link from its initial configuration to any configuration that satisfies $(\pi/2 - \alpha) \leq \theta_1 \leq (\pi/2 + \alpha)$, $\dot\theta_1 = 0$, as shown in Fig.4.

3. Swing-up control of the second link:

If the configuration of the first link satisfies $(\pi/2 - \alpha) \leq \theta_1 \leq (\pi/2 + \alpha)$, $\dot\theta_1 = 0$, the second link will automatically swing up to the configuration $(\theta_2, \dot\theta_2) \approx (0, 0)$ if $E_2 \approx E_{2T}$. To increase the energy of the second link to E_{2T}, we will use a series of rest-to-rest maneuvers of the first link, described in section 3.2. Additionally, to ensure that θ_1 will always lie within the region $(\pi/2-\alpha) \leq \theta_1 \leq (\pi/2+\alpha)$, the following procedures will be followed:

To hold the first link fixed, the holding torque τ_h in Eq.(13) will be applied. To move θ_1 in the positive (counter-clockwise) direction from rest, the torque expression in Eq.(30) will be used provided it is greater than τ_h at the initial time. To move θ_1 in the negative (clockwise) direction from rest, the torque expression in Eq.(30) will be used provided it is less than τ_h at the initial time. As θ_1 approaches the boundary of the interval $[(\pi/2 - \alpha), (\pi/2 + \alpha)]$, the braking torque τ_b in Eq.(18) will be used; a large value of k_1 will be used to quickly stop the motion of the first link.

4. Stabilization:

With $(\pi/2 - \alpha) \leq \theta_1 \leq (\pi/2 + \alpha)$, $\dot\theta_1 = 0$, and $E_2 \approx E_{2T}$, the second link will behave like a pendulum and will reach the vertically upright configuration in finite time. Concurrently, the configuration of the system will reach the region of attraction of the desired equilibrium $(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (\pi/2, 0, 0, 0)$. Invoke the linear controller, designed in the first step of the algorithm, to stabilize the desired equilibrium when $(\theta_2, \dot\theta_2) \approx (0, 0)$.

V. Numerical Simulations

The kinematic and dynamic parameters of the pendubot were assumed to be

$$l_i = 1.0\ m, \quad d_i = 0.5\ m, \quad m_i = 1.0\ kg, \quad I_i = \frac{1}{12} m_i l_i^2 = 0.0833\ kgn$$

For this choice of parameters, E_{2T} was evaluated to be $14.715\ J$.

Impulse-momentum model of braking: In section 3.1 we modeled sudden stopping of the first link by the action of an impulsive force and an impulsive moment on the second link. Here we show that this modeling assumption is accurate for large values of gain k_1 in the expression for the braking torque in Eq.(18), which we know will cause sudden stopping of the first link. We consider the pendubot configuration $(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (0.0, 2.0, 0.0, 3.0)$, where the units are rad and rad/s. If the first joint is stopped instantaneously, the velocity of the second joint and change in energy of the second link can be computed using Eqs.(24) and (26), respectively. Specifically, using $\dot\theta_1^- = 2.0\ rad/s$ and $\dot\theta_2^- = 3.0\ rad/s$, these values can be computed as

$$\dot\theta_2^+ = 8.0\ rad/s, \qquad \Delta E_2 = -0.5\ J \tag{31}$$

TABLE I

Comparison of simulation results of braking with analytical results of sudden stopping

k_1	10	100	1000
$\Delta t\ (sec)$	0.800	0.080	0.008
$\Delta\theta_1\ (rad)$	0.200	0.020	0.002
$\dot\theta_2^+\ (rad/s)$	5.890	7.330	7.899
difference (%)	26.38	8.37	1.26
$\Delta E_2\ (J)$	0.724	0.478	0.495
difference (%)	44.76	4.30	0.92

The values of $\dot\theta_2^+$ and ΔE_2, obtained from simulations, are tabulated above for different values of gain k_1 used in the expression for the braking torque in Eq.(18). It is clear that the difference of these values from those in Eq.(31) are negligible for large values of gain k_1. Also, as expected, large values of k_1 require less time for the first link to come to rest and small angle of travel of the first link before it comes to rest. In our simulation of swing-up control, presented next, we used a moderate value of $k_1 = 100$.

Swing-up control and stabilization: As part of the initialization (first step of the algorithm), a linear controller is designed to stabilize the desired equilibrium. Through repeated simulation of the closed-loop system behavior, α is estimated to be 10 deg. The initial configuration of the pendubot is chosen as

$$(\theta_1, \dot\theta_1, \theta_2, \dot\theta_2) = (90.0, 0.0, -135.0, 0.0) \tag{32}$$

where the units are deg and deg/s. This choice of initial configuration eliminates the need for the second step of the

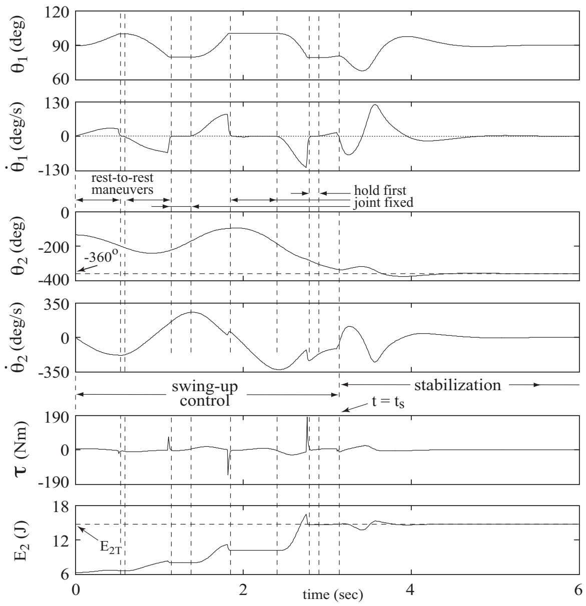

Fig. 5. Plot of joint angles, joint angle velocities, control torque, and energy of second link

algorithm, which is trivial. The simulation results for the third and fourth steps of the algorithm are shown in Fig.5; the plots show the two joint angles, their velocities, the control torque and the energy of the second link.

It can be seen from Fig.5 that E_2 reaches the value of E_{2T} at $t = t_s = 3.15\ sec$. Since the second link is close to its vertically upright configuration at this time, the linear controller is invoked immediately for stabilization. The swing-up control of the second link is achieved over the interval $t \in [0, 3.15]\ sec$ through a series of rest-to-rest maneuvers separated by periods of time over which the first joint is held fixed. It can be seen from Fig.5 that E_2 increases for each rest-to-rest maneuver but remains constant during times when the first link is held fixed. The increase of E_2 during each rest-to-rest maneuver is achieved through positive work done by the first link followed by energy loss due to braking. During braking, the control torque peaks. The large magnitude of these peaks, which can be attributed to the large value of gain k_1, is not of concern since they are applied intermittently and for short time intervals.

VI. Conclusion

This paper presents a new solution to the swing-up control problem of the pendubot. The solution is based on taking the first link to its vertically upright position and executing a series of rest-to-rest maneuvers about this position with a small amplitude of oscillation. Using the principles of work-energy and impulse and momentum, the rest-to-rest maneuvers are designed to increase the energy of the second link. The rest-to-rest maneuvers are carried out till the energy of the second link equals its maximum potential energy. This results in the second link swinging up to its vertically upright position and the pendubot reaching a configuration from which the desired equilibrium can be stabilized using a linear controller. Simulation results are presented to demonstrate the feasibility of the proposed approach. Our future work will explore the feasibility of extending our approach to swing-up control of the acrobot and control of other under-actuated robotic systems.

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