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SWING-UP CONTROL OF THE PENDUBOT THROUGH ENERGY MANAGEMENT OF THE UNDERACTUATED LINK

Thamer Albahkali
Graduate Student

Ranjan Mukherjee*
Professor

Tuhin Das
Visiting Assistant Professor

Department of Mechanical Engineering
Michigan State University
East Lansing, Michigan 48824
mukherji@egr.msu.edu

ABSTRACT

The control problem of the pendubot refers to the task of stabilizing its equilibrium configuration with the highest potential energy. Linearization of the dynamics of the pendubot about this equilibrium results in a completely controllable system and allows a linear controller to be designed for local asymptotic stability. Therefore, the real challenge is to design a control law for the underactuated system that will swing up both the links and brings the configuration variables of the system within the region of attraction of the equilibrium. This paper provides a method for swing-up control based on a series of start and stop maneuvers of the first link about its vertically upright configuration. The start and stop maneuvers are designed such that each maneuver results in a net gain in the energy of the second link. This results in swinging up of the second link and the pendubot configuration reaching the region of attraction of the desired equilibrium. A three step algorithm is provided for swing up control followed by stabilization. Simulation results are presented to demonstrate the efficacy of the approach.

NOMENCLATURE

m_i	mass of the i-th link, kg
l_i	length of the i-th link, m
d_i	distance between the center of mass of the i-th link and the i-th joint, m
I_i	mass moment of inertia of the i-th link about its center of mass, $kg\,m^2$
θ_i	angle of the i-th link as shown in Fig.1, rad
g	acceleration due to gravity, 9.81 m/s^2
τ	external torque applied at the first joint, $N\,m$
F	force at the second joint that does positive work on the second link, N
E_2	total energy of the second link, $kg\,m^2/s^2$
E_{2T}	PE_2 when $\theta_1 = \pi/2$ and $\theta_2 = 0$, $kg\,m^2/s^2$
S_i	$\sin\theta_i$
C_i	$\cos\theta_i$
S_{12}	$\sin(\theta_1+\theta_2)$
C_{12}	$\cos(\theta_1+\theta_2)$

*Address all correspondence to this author.

1 INTRODUCTION

The Pendubot [2, 8] is a two-link planar robot in the vertical plane with an actuator at the shoulder joint but no actuator at the elbow joint. The control problem of the pendubot typically refers to the task of stabilizing its equilibrium point with the highest potential energy, which is unstable. It is a classic example of an underactuated system [9] and its control problem has similarities with the control problem of the single and double inverted pendulums on a cart, the planar underactuated robot [1], and the Acrobot [3].

The overall control problem of the pendubot requires swing-up control that brings the configuration of the pendubot close to its equilibrium configuration with the highest potential energy followed by balance control that stabilizes the equilibrium. The balance control problem was addressed by several researchers. Among them, Spong and Block [8] linearized the dynamic equations and used linear quadratic regulator theory for pole placement and Zhang and Tarn [10] used hybrid control methods. In this paper we linearize the dynamic equations and use a linear controller for balance control like Spong and Block [8], but our main contribution lies in swing-up control of the pendubot.

For the swing-up control problem, Spong and Block [8] proposed feedback linearization techniques and Fantoni, et al. [4] provided an energy based control method together with stability analysis. More recently, Grognard and Canudas-de-Wit [6] and Orlov, et al. [7] have proposed methods based on zero dynamics of the system that result in limit cycle behavior. Our approach to swing-up control is similar to the energy based approach of Fantoni, et al. [4] but we focus on the force interaction between the two links at the underactuated joint and the change in energy of the second link resulting from it. Our method is based on a series of start and stop maneuvers of the first link about its vertically upright configuration that results in swinging-up of the second link and the pendubot configuration reaching the region of attraction of the desired equilibrium point. At that juncture the controller can be switched to stabilize the equilibrium.

This paper is organized as follows. The equations of motion of the pendubot and the force of the interaction between the two links are provided in Section 2. In Section 3 we design a start-and-stop maneuver of the first link that results in net gain in energy of the second link. The algorithm for swing-up control followed by stabilization of the desired equilibrium configuration is discussed in Section 4. Section 5 provides simulation results and Section 6 contains concluding remarks.

2 SYSTEM DYNAMICS

2.1 Equation of Motion

Consider the pendubot in Fig.1. Assuming an ideal system with no friction in the joints, the equation of motion can be obtained using the Lagrangian formulation as follows [1]

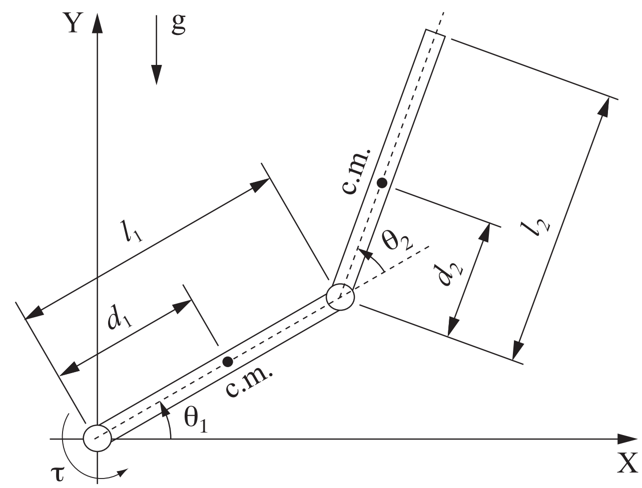

Figure 1. THE PENDUBOT

$$A(\theta)\ddot{\theta} + B(\theta,\dot{\theta})\dot{\theta} + G(\theta) = T \quad (1)$$

where

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \qquad T = \begin{pmatrix} \tau \\ 0 \end{pmatrix} \quad (2)$$

and

$$A(\theta) = \begin{pmatrix} q_1 + q_2 + 2q_3C_2 & q_2 + q_3C_2 \\ q_2 + q_3C_2 & q_2 \end{pmatrix} \quad (3)$$

$$B(\theta) = q_3S_2 \begin{pmatrix} -\dot{\theta}_2 & -(\dot{\theta}_1 + \dot{\theta}_2) \\ -\dot{\theta}_1 & 0 \end{pmatrix} \quad (4)$$

$$G(\theta) = g \begin{pmatrix} q_4C_1 + q_5C_{12} \\ q_5C_{12} \end{pmatrix} \quad (5)$$

In Eqs.(3), (4) and (5), q_i, $i = 1, 2, \cdots, 5$ are constants and have the following expressions [1]

$$\begin{aligned} q_1 &= m_1d_1^2 + m_2l_1^2 + I_1 \\ q_2 &= m_2d_2^2 + I_2 \\ q_3 &= m_2l_1d_2 \\ q_4 &= m_1d_1 + m_2l_1 \\ q_5 &= m_2d_2 \end{aligned} \quad (6)$$

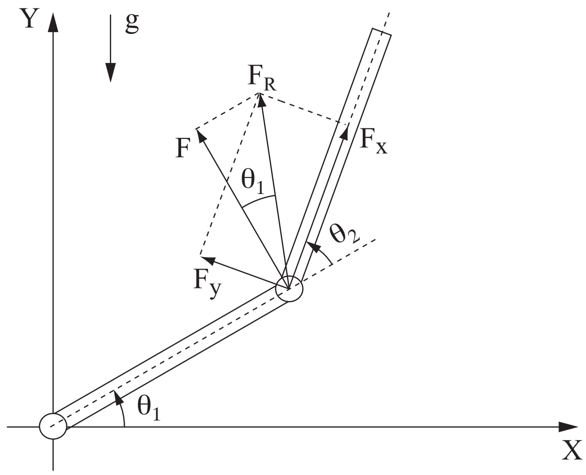

Figure 2. FORCES OF INTERACTION BETWEEN THE LINKS

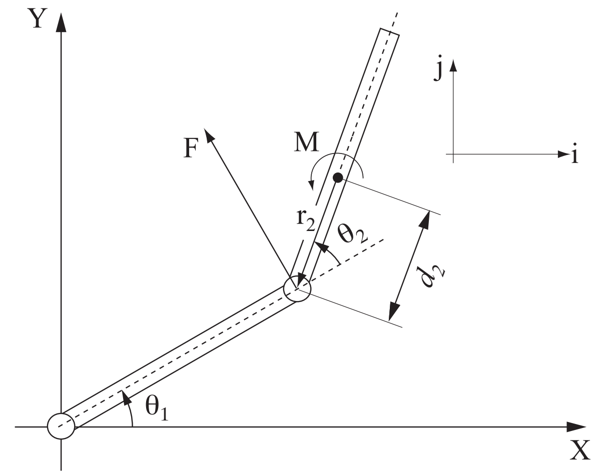

Figure 3. EFFECT OF SUDDEN STOPPING OF THE FIRST LINK

2.2 Force of Interaction Between the Two Links

By applying the Newton-Euler method [5], the force of interaction between the two links can be expressed as follows:

$$\begin{aligned} F_x &= m_2\left\{-d_2(\dot{\theta}_1+\dot{\theta}_2)^2 + l_1(\ddot{\theta}_1 S_2 - \dot{\theta}_1^2 C_2) + g S_{12}\right\} \\ F_y &= m_2\left\{d_2(\ddot{\theta}_1+\ddot{\theta}_2) + l_1(\ddot{\theta}_1 C_2 + \dot{\theta}_1^2 S_2) + g C_{12}\right\} \end{aligned} \tag{7}$$

where the directions of F_x and F_y are shown in Fig.2. The resultant of F_x and F_y is F_R. F_R can be decomposed into a workless constraint force along the length of the first link and the component F that does work on link 2. F can be expressed in terms of F_x and F_y as follows

$$\begin{aligned} F &= F_x S_2 + F_y C_2 \\ &= m_2\left\{l_1\ddot{\theta}_1 + d_2(\ddot{\theta}_1+\ddot{\theta}_2)C_2 - d_2(\dot{\theta}_1+\dot{\theta}_2)^2 S_2 + g C_1\right\} \end{aligned} \tag{8}$$

In addition, one can look into the expression of E_2 and its derivative as follows

$$E_2 = \frac{1}{2} I_2(\dot{\theta}_1+\dot{\theta}_2)^2 + \frac{1}{2} m_2 \vec{v}_2 \cdot \vec{v}_2 + m_2\, g(l_1 S_1 + d_2 S_{12}) \tag{9}$$

where $\vec{v}_2$ is the velocity vector of the center of mass of the second link and is given by the expression

$$\vec{v}_2 = -\left[l_1\,\dot{\theta}_1 S_1 + d_2(\dot{\theta}_1+\dot{\theta}_2)S_{12}\right]\vec{i} + \left[l_1\,\dot{\theta}_1 C_1 + d_2(\dot{\theta}_1+\dot{\theta}_2)C_{12}\right]\vec{j} \tag{10}$$

By differentiating the expression for E_2, we get

$$\dot{E}_2 = m_2 l_1 \dot{\theta}_1\left\{l_1\ddot{\theta}_1 + d_2(\ddot{\theta}_1+\ddot{\theta}_2)C_2 - d_2(\dot{\theta}_1+\dot{\theta}_2)^2 S_2 + g C_1\right\} \tag{11}$$

It can be verified that $\dot{E}_2 = F\,(l_1\dot{\theta}_1)$, where $(l_1\dot{\theta}_1)$ is the velocity of the point of application of the force F and have the same direction as that of F.

3 SECOND LINK ENERGY CONSIDERATIONS

3.1 Effect of Sudden Stopping of the First Link

The action of suddenly stopping the first link has the effect of application of an impulsive force and an impulsive moment on the second link, as shown in Fig.3. The impulsive force results in a change in the linear momentum of the second link and the impulsive moments results in a change in its angular momentum. The change in the linear and angular momentum of the second link can be expressed as follows

$$\vec{F}\,\Delta t = m_2\,(\vec{v}_2^{\,+} - \vec{v}_2^{\,-}) \tag{12}$$

$$\vec{M}\,\Delta t = \vec{r}_2 \times \vec{F}\,\Delta t = I_2\,\dot{\theta}_2^{+} - I_2\,(\dot{\theta}_1^{-} + \dot{\theta}_2^{-}) \tag{13}$$

where Δt is the interval of time over which the impulsive force and impulsive moment act, $\vec{v}_2^{\,+}$ and $\vec{v}_2^{\,-}$ are the velocity vectors of the center of mass of the second link before and after the action of the impulsive force, respectively, and $\dot{\theta}_2^{+}$ and $\dot{\theta}_2^{-}$ are the angular velocities of the second link before and after the action of the impulsive moment, respectively. The angular velocity of the first link, before it is suddenly stopped is denoted by $\dot{\theta}_1^{-}$. The expressions for $\vec{v}_2^{\,+}$, $\vec{v}_2^{\,-}$, and $\vec{r}_2$ can be written as follows

$$\begin{aligned} \vec{v}_2^{\,+} &= d_2\,\dot{\theta}_2^{+}\,(-S_{12}\vec{i} + C_{12}\vec{j}) \\ \vec{v}_2^{\,-} &= -\left[l_1\,\dot{\theta}_1^{-} S_1 + d_2(\dot{\theta}_1^{-} + \dot{\theta}_2^{-})S_{12}\right]\vec{i} \end{aligned} \tag{14}$$

$$+\left[l_1\dot{\theta}_1^- C_1 + d_2(\dot{\theta}_1^- + \dot{\theta}_2^-)C_{12}\right]\vec{j} \quad (15)$$

$$\vec{r}_2 = -d_2(C_{12}\vec{i} + S_{12}\vec{j}) \quad (16)$$

where $\vec{i}$ and $\vec{j}$ are unit vectors, as shown in Fig.3. By substituting Eqs.(14), (15) and (16) into Eq.(13), we get

$$\dot{\theta}_2^+ = \dot{\theta}_2^- + \left[1 + \frac{l_1 m_2 d_2 C_2}{I_2 + m_2 d_2^2}\right]\dot{\theta}_1^- \quad (17)$$

Since there is no change in the potential energy of the second link over the Δt time interval, the change in the total energy of the second link is due to the change in its kinetic energy alone, and is equal to

$$\Delta E_2 = \frac{1}{2}(I_2 + m_2 d_2^2)(\dot{\theta}_2^+)^2 - \frac{1}{2}I_2\left[(\dot{\theta}_1^-)^2 + (\dot{\theta}_2^-)^2\right] - \frac{1}{2}m_2\left\{l_1^2(\dot{\theta}_1^-)^2 + d_2^2(\dot{\theta}_1^- + \dot{\theta}_2^-)^2 + 2l_1 d_2\dot{\theta}_1^-(\dot{\theta}_1^- + \dot{\theta}_2^-)C_2\right\} \quad (18)$$

By substituting Eq.(17) into Eq.(18), we can express the change in the total energy of the second link in terms of the velocity of the first link prior to stopping, as follows

$$\Delta E_2 = \frac{1}{2}m_2 l_1^2\left[\frac{m_2 d_2^2 C_2^2}{I_2 + m_2 d_2^2} - 1\right](\dot{\theta}_1^-)^2 \quad (19)$$

Since $m_2 d_2^2 C_2^2 < (I_2 + m_2 d_2^2)$, $\Delta E_2 \leq 0$ and $\Delta E_2 = 0$ if only if $\dot{\theta}_1^- = 0$. Clearly, the total energy of the second link decreases whenever the first link is suddenly stopped.

3.2 Effect of A Start and Stop Manuever

Consider the pendubot in the initial configuration shown in Fig.4, where $\theta_1 = \pi/2 - \alpha$ (α is a small angle), $\dot{\theta}_1 = 0$, and the second link is moving freely. Now consider the application of a torque τ that takes θ_1 from $\pi/2 - \alpha$ to $\pi/2 + \alpha$ and then stops the first link. The net work done on the second link is equal to

$$\Delta E_2 = \int_{\pi/2-\alpha}^{\pi/2+\alpha} F\, l_1 d\theta_1 + \frac{1}{2}m_2 l_1^2\left[\frac{m_2 d_2^2 C_2^2}{I_2 + m_2 d_2^2} - 1\right](\dot{\theta}_1^-)^2 \geq l_1\int F\,\dot{\theta}_1 dt - \frac{1}{2}m_2 l_1^2(\dot{\theta}_1^-)^2 \quad (20)$$

where $\dot{\theta}_1^-$ is the velocity of the first link before it is stopped and F is given by the expression in Eq.(8). If we now impose the constraint

$$d_2(\ddot{\theta}_1 + \ddot{\theta}_2)C_2 - d_2(\dot{\theta}_1 + \dot{\theta}_2)^2 S_2 + gC_1 = k\,l_1\ddot{\theta}_1, \quad k > 0 \quad (21)$$

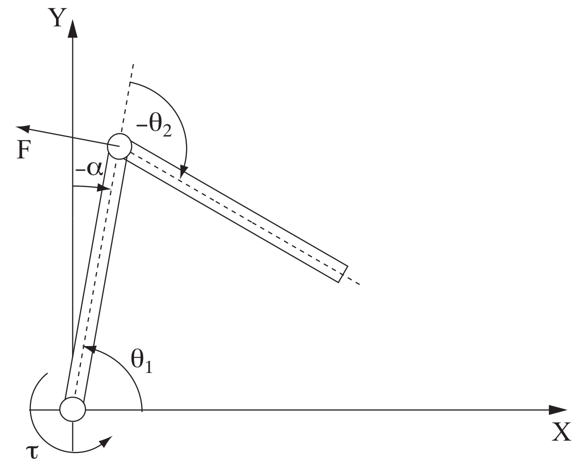

Figure 4. EFFECT OF A START AND STOP MANEUVER

we get from Eq.(20)

$$\begin{aligned}\Delta E_2 &\geq l_1\int F\,\dot{\theta}_1 dt - \frac{1}{2}m_2 l_1^2(\dot{\theta}_1^-)^2 \\ &= l_1\int m_2\left[(1+k)l_1\ddot{\theta}_1\right]\dot{\theta}_1 dt - \frac{1}{2}m_2 l_1^2(\dot{\theta}_1^-)^2 \\ &= \left[\frac{1+k}{2}\right]m_2 l_1^2\int 2\dot{\theta}_1\ddot{\theta}_1\, dt - \frac{1}{2}m_2 l_1^2(\dot{\theta}_1^-)^2 \\ &= \left[\frac{1+k}{2}\right]m_2 l_1^2(\dot{\theta}_1^-)^2 - \frac{1}{2}m_2 l_1^2(\dot{\theta}_1^-)^2 \\ &= \frac{1}{2}k\,m_2 l_1^2(\dot{\theta}_1^-)^2 > 0\end{aligned} \quad (22)$$

Using Eq.(1) it can be shown that the constraint in Eq.(21) can be imposed by application of the torque

$$\tau = \left[\frac{q_1 q_2 - q_3^2 C_2^2}{k l_1 q_2 + d_2 q_3 C_2^2}\right]\left\{gC_1 - d_2(\dot{\theta}_1 + \dot{\theta}_2)^2 S_2\right\} + \left[\frac{1}{k l_1 q_2 + d_2 q_3 C_2^2}\right]\left\{d_2(h_1 + h_2)C_2 - k l_1 h_1\right\} \quad (23)$$

where h_1 and h_2 are given by the expressions

$$h_1 = q_2 q_3(\dot{\theta}_1 + \dot{\theta}_2)^2 S_2 + q_3^2\dot{\theta}_1^2 S_2 C_2 + g(q_3 q_5 C_2 C_{12} - q_2 q_4 g C_1) \quad (24)$$

$$h_2 = -(\dot{\theta}_1 + \dot{\theta}_2)^2(q_2 q_3 + q_3^2 C_2)S_2 - (q_1 + q_3 C_2)q_3\dot{\theta}_1^2 S_2 - g\left\{q_3 q_5 C_2 C_{12} + (q_2 + q_3 C_2)q_4 C_1 - q_1 q_5 C_{12}\right\} \quad (25)$$

Clearly, the net energy of the second link will increase if the first link is driven from $\theta_1 = \pi/2 - \alpha$ to $\theta_1 = \pi/2 + \alpha$ using the torque expression in Eq.(23), and then stopped.

4 ALGORITHM FOR SWING-UP CONTROL

Based on the results in the last section, a simple three-step algorithm can be proposed for control of the pendubot:

1. **Initialization:** Drive the first link from its initial configuration to $\theta_1 = \pi/2 - \alpha$, $\dot{\theta}_1 = 0$, where α is a small number (positive or negative) chosen such that $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) = (\pi/2 \pm \alpha, 0, 0, 0)$ lies in the region of attraction of the equilibrium point $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) = (\pi/2, 0, 0, 0)$.

2. **Swing-Up Control:** Using the torque expression in Eq.(23), drive the first link from $\theta_1 = \pi/2 - \alpha$ to $\theta_1 = \pi/2 + \alpha$ and stop. From the last section we know that this will add energy to the second link. Using the torque expression in Eq.(23), drive the first link from $\theta_1 = \pi/2 + \alpha$ to $\theta_1 = \pi/2 - \alpha$ next, and stop. Repeat the procedure of driving the first link between $\pi/2 \pm \alpha$ till $E_2 \approx E_{2T}$.

3. **Stabilization:** Maintain the configuration of the first link at $\theta_1 = \pi/2 - \alpha$ or $\theta_1 = \pi/2 + \alpha$ and $\dot{\theta}_1 = 0$, and wait till the second link configuration reaches $\theta_2 = 0$, $\dot{\theta}_2 \approx 0$ or $\dot{\theta}_2 = 0$, $\theta_2 \approx 0$. This will happen in finite time since $E_2 \approx E_{2T}$. Switch to a linear controller based on linearized dynamics of the plant about the equilibrium point $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) = (\pi/2, 0, 0, 0)$.

5 SIMULATION

In our simulations, the kinematic and dynamics parameters of the pendubot were chosen as follows:

$$l_1 = 1.0, \quad l_2 = 1.0, \quad d_1 = 0.5, \quad d_2 = 0.5$$

$$m_1 = 0.1, \quad m_2 = 0.1$$

$$I_1 = m_1 l_1^2/12, \quad I_2 = m_2 l_2^2/12 \tag{26}$$

where SI units were used. From these parameter values, E_{2T} was computed to be 1.4715 J. The initial conditions of the pendubot were assumed to be

$$\theta_1(0) = 1.55, \;\; \dot{\theta}_1(0) = 0.0, \;\; \theta_2(0) = -0.75\pi, \;\; \dot{\theta}_2(0) = 0.0 \tag{27}$$

where the units were rad and rad/sec for angular position and angular velocity, respectively. The value of k in Eq.(21) was chosen as unity. The simulation results are shown in Fig.5.

The initial conditions of the pendubot satisfies $\theta_1 = \pi/2 - \alpha$ with $\alpha = 0.02$ rad and this precludes the need for the first step of the algorithm. The second step of the algorithm is initiated at time $t = 0$ sec. Between $t = 0$ and $t = t_s \approx 20.3$ sec, the first link executes several start and stop maneuvers about the mean position of $\theta_1 = \pi/2$ to systematically increase the energy of the second link. The position of the first and second links are shown in Figs.5(a) and 5(b), respectively. The input torque and the energy of the second link are shown in Figs.5(c) and 5(d), respectively. It is clear from Fig.5(d) that each start and stop maneuver results in a net gain in energy of the second link. After the energy of the second link equals E_{2T} and the pendubot configuration reaches the region of attraction of the desired equilibrium configuration, a linear controller is invoked at time $t = t_s$ sec. It can be seen that the desired equilibrium configuration is stabilized after a brief transient. The linear controller was designed using simple pole placement techniques and is therefore not provided here.

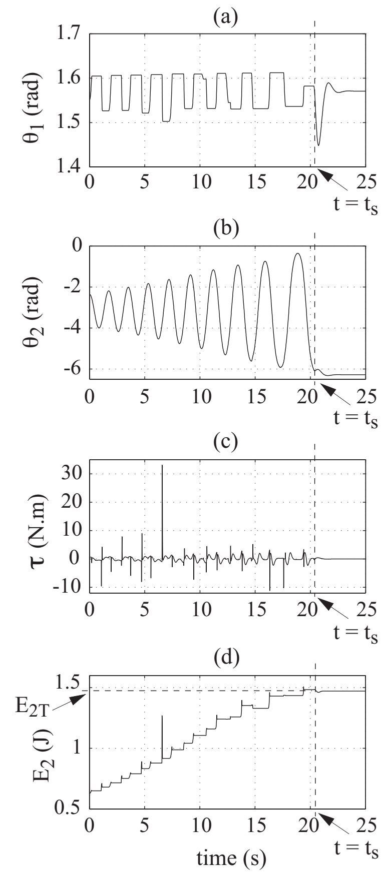

Figure 5. SIMULATION RESULTS

6 CONCLUSION

This paper presents a new solution to the swing-up control problem of the pendubot. The solution is based on taking the first link to its vertically upright position and executing a series of start and stop maneuvers about this position with a small amplitude of oscillation. Using the principles of work-energy and impulse and momentum, each start and stop maneuver is designed to increase the total energy of the second link. This ultimately results in the second link swinging up to its vertically upright position and the pendubot reaching a configuration within the region of attraction of the desired equilibrium. A linear controller can then be invoked to stabilize the equilibrium configuration. Simulation results are presented to demonstrate the feasibility of the proposed approach. Our future work will explore the feasibility of extending our approach to stabilization of the acrobot.

REFERENCES

[1] Arai, H., and Tachi, S. "Position Control of a Manipulator with Passive Joints Using Dynamic Coupling," IEEE Trans. on Robotics and Automation, Vol.7, pp.528-534, 1991

[2] Block, D. J. "Mechanical Design and Control of the Pendubot," MS Thesis, Department of General Engineering, University if Illinois at Urbana-Champaign, 1996

[3] Bortoff, S. A. "Pseudolinearization Using Spline Functions with Application to the Acrobot," PhD Dissertation, Department of Electrical and Computer Engineering, University if Illinois at Urbana-Champaign, 1992

[4] Fantoni, I., Lozano, R., and Spong, M. W. "Energy Based Control of the Pendubot," IEEE Trans. on Automatic Control, Vol.45, No.4, pp.725-729, 2000

[5] Fu, K. S., Gonzales, R. C., and Lee, C. S. G. "Robotics: Control, Sensing, Vision, and Intelligence," McGraw Hill, New York, NY, 1987.

[6] Grognard, F., and Canudas-de-Wit, C. "Virtual Constraints for the Orbital Stabilization of the Pendubot," Nonlinear and Adaptive Control: Theory and Algorithms for the User, Ed. A. Astolfi, Imperial College Press, pp.115-145, 2005

[7] Orlov, Y., Aguilar, L. T., and Acho, L. "Model Orbit Robust Stabilization (MORS) of Pendubot with Application to Swing Up Control," 44th IEEE International Conference on Decision and Control, Seville, Spain, pp.6164-6169, 2005

[8] Spong, M. W., and Block, D. J. "The Pendubot: A Mechatronic System for Control Research and Education," 34th IEEE International Conference on Decision and Control, New Orleans, pp.555-557, 1995

[9] Spong, M. W. "Underactuated Mechanical Systems," Control Problems in Robotics and Automation, B. Siciliano and K. P. Valavanis Eds., LNCIS, Vol.230, pp.135–150, Springer Verlag, London, 1998

[10] Zhang, M., and Tarn, T. -J. "Hybrid Control of the Pendubot," IEEE/ASME Transactions on Mechatronics, Vol.7, No.1, pp.79-86, 2002